## STAT 479 Quiz 3 Fall 2022 October 20, 2022

1. You have two observations from a Pareto distribution with parameters of  $\alpha$  = 2 and  $\theta$  .

The first observation is 500 and the second observation is greater than 500 but you do not know the value.

Using this information, calculate the Maximum Likelihood Estimator of  $\,\theta\,$  .

## Solution:

$$L(\theta) = f(500) \left[ 1 - F(500) \right] = \left[ \frac{2\theta^2}{(500 + \theta)^3} \right] \left[ \frac{\theta}{500 + \theta} \right]^2 = \frac{2\theta^4}{(500 + \theta)^5} = 2\theta^4 \left[ (500 + \theta)^{-5} \right]$$

$$l(\theta) = \ln(2) + 4\ln(\theta) - 5\ln(500 + \theta)$$

$$l'(\theta) = 0 + \frac{4}{\theta} - \frac{5}{500 + \theta} = 0$$

$$\frac{4}{\theta} = \frac{5}{500 + \theta} = 34(500 + \theta) = 5\theta = 32000 + 4\theta = 5\theta = 32000$$

2. You are given the following observations from a uniform distribution on the range of 0 to U:

3, 5, 8, and 12.

Calculate the Maximum Likelihood Estimator of  $\boldsymbol{U}\,$  .

## Solution:

 $U = Max\{3, 5, 8, 12\} = 12$