STAT 479
Test 2
Fall 2022

November 1, 2022

1. The number of traffic tickets in a year is assumed to be distributed as a Poisson distribution with a parameter of λ.

   During the last year, the number of traffic tickets to the students in this class have been:

Number of Tickets	Number of Students
0	8
1	10
2	4
3	2
6	1
Total	25

   a. (3 points) Calculate the value of $\hat{\lambda}$ which is the Maximum Likelihood Estimator of λ.

      Solution:

$$\hat{\lambda} = \bar{X} = \frac{(0)(8)+(1)(10)+(2)(4)+(3)(2)+(6)(1)}{25} = 1.2$$

   b. (7 points) Calculate the 80% confidence interval for λ.

      Solution:

$$Var[\hat{\lambda}] = \frac{\hat{\lambda}}{n} = \frac{1.2}{25} = 0.048$$

$$CI = \hat{\lambda} \pm 1.282\sqrt{0.048} = (0.91912783;1.480872127)$$

2. (10 points) For dental claims, it is assumed that claims are distributed as a normal distribution with parameters of μ and σ .

   You have the following sample of claims:

   X : 150 200 250 300 300 300 400 400 500 1000

   $$\sum X_i = 3800 \qquad \sum X_i^2 = 1,965,000$$

   Calculate the Maximum Likelihood Estimators for the parameters.

   Solution:

$$\hat{\mu} = \bar{X} = \frac{3800}{10} = 380$$

$$\hat{\sigma} = \sqrt{\frac{(150-380)^2 + (200-380)^2 + (250-380)^2 + 3(300-380)^2 + 2(400-380)^2 + (500-380)^2 + (1000-380)^2}{10}}$$

$$= 228.254$$

or

$$\hat{\sigma} = \sqrt{\frac{1,965,000-(10)(380)^2}{10}} = 228.254$$

3. (15 point) The random variable X is distributed as a Weibull distribution with parameters $\tau = 2$ and θ .

   We have two observations of X which are 5 and 10.

   Calculate the Maximum Likelihood Estimator of θ.

   Solution:

$$L(\theta) = f(5) \cdot f(10)$$

$$= \frac{2\left(\frac{5}{\theta}\right)^2 e^{-(5/\theta)^2}}{5} \frac{2\left(\frac{10}{\theta}\right)^2 e^{-(10/\theta)^2}}{10} = \frac{4\left(\frac{2500}{\theta^4}\right)e^{-125/\theta^2}}{50} = \frac{200}{\theta^4} e^{-125/\theta^2}$$

$$l(\theta) = \ln(200) - 4\ln(\theta) - 125\theta^{-2}$$

$$l'(\theta) = 0 - \frac{4}{\theta} - 125(-2)\theta^{-3} = 0 ==> -4\theta^2 + 250 = 0$$

$$\theta = \sqrt{\frac{250}{4}} = 7.90569$$

4. (5 points) The future lifetime of mice is assumed to be a uniform distribution between zero and U .

The following data has been observed by following the lives of 100 mice:

Days Lived	**Number of Mice**
0 – 2	10
3 – 8	40
8 – X	30

The rest of the mice live longer than X days.

The Maximum Likelihood Estimator for U is 15 days.

Determine X .

Solution:

X is the censoring point.

$$\hat{U} = 15 = (\text{Censoring Point})\left(\frac{\text{Total Mice}}{\text{Number of Mice Below Censoring Point}}\right)$$

$$15 = X\left(\frac{100}{80}\right) ==> X = 12$$

5. Kiley buys automobile insurance coverage from Lupo Insurance Company. Over the next 8 years, Kiley has claims of:

   0, 0, 400, 500, 1000, 1300, 2000, 2000

The number of claims is assumed to be distributed as a Poisson distribution.

a. (5points) Calculate the criterion for full credibility (the number of claims) for the amount of claims in this situation to be within 8% of the true amount of claims at least 70% of the time.

   Solution:

$$\Phi(y_0) = \frac{1+P}{2} = \frac{1.70}{2} = 0.85 ==> y_0 = 1.036$$

$$n = \lambda_0 = \left(\frac{1.036}{0.08}\right)^2 = 167.7025 \text{ which is the answer since frequency is Poisson}$$

b. (5 points) Calculate the criterion for full credibility (the number of claims) for the pure premium in this situation to be within 8% of the true value of the pure premium at least 70% of the time.

   Solution:

$$n = \lambda_0\left(1+\left(\frac{\sigma_X}{\mu_X}\right)^2\right)$$

$$\mu_X = \frac{0+0+400+500+1000+1300+2000+2000}{8} = 900$$

$$\sigma_X = \sqrt{\frac{(2)(0-900)^2+(400-900)^2+(500-900)^2 + (1000-900)^2+(1300-900)^2+2(2000-900)^2}{8-1}}$$

$$= 812.40384$$

$$n = (167.7025)\left(1+\left(\frac{812.40384}{900}\right)^2\right) = 304.34898$$

c. (5 points) Kiley's manual premium is 1000. Using partial credibility, determine the premium that Lupo should charge Kiley in the next year.

Solution:

$$Z = \sqrt{\frac{8}{304.34898}} = 0.16213$$

$$\Pr emium = (0.16213)(900) + (1 - 0.16213)(1000) = 983.79$$

6. You are given:
   (i) The number of claims has a Negative Binomial distribution with parameters of γ and $\beta = 2$.
   (ii) Claim amounts uniformly distributed between 100 and 200.
   (iii) The number of claims and claim sizes are independent.
   (iv) The observed value should be within 5% of the expected value 98% of the time.

a. (5 points) Calculate the expected number of claims needed for full credibility for the frequency.

Solution:

$$\Phi(y_0) = \frac{1+P}{2} = \frac{1.98}{2} = 0.99 ==> y_0 = 2.326$$

$$\lambda_0 = \left(\frac{2.326}{0.05}\right)^2 = 2164.11$$

$$n = \lambda_0 \left(\frac{\sigma_N^2}{\mu_N}\right) = (2164.11)\left(\frac{2\gamma(1+\beta)}{2\gamma}\right) = (2164.11)(1+\beta) = (2164.11)(3) = 6492.33$$

b. (5 points) Calculate the expected number of claims needed for full credibility for the severity.

Solution:

$$n = \lambda_0 \left(\frac{\sigma_X}{\mu_X}\right)^2 = (2164.11)\left(\frac{833.33}{150^2}\right) = 80.1522$$

$$\mu_X = \frac{100+200}{2} = 150 \qquad \sigma_X^2 = \frac{(200-100)^2}{12} = 833.33$$

c. (5 points) Calculate the expected number of claims needed for full credibility for the pure premium.

Solution:

$$n = 6492.33 + 80.15 = 6572.48$$

d. (4 points) If you had 6,000 claims, what would Z be for pure premium credibility?

Solution:

$$Z = \min\left(1, \sqrt{\frac{6000}{6572.48}}\right) = 0.95546$$

7. Automobile insurance consists of five coverages which are listed below in alphabetic order:

    i. Collision
    ii. Liability Insurance
    iii. Medical Benefits
    iv. Other than collision (Comprehensive)
    v. Uninsured and underinsured motorists

    a. (2 points) State the coverage(s) that are required by law.

       Solution:

       Liability and Medical

    b. (2 points) State the coverage(s) that are required if you have purchased your car with using a loan.

       Solution:

       Collision and Other Than Collision

    c. (2 point) State the coverage(s) that provide coverage when a rock breaks your windshield.

       Solution:

       Other Than Collision

8. (8 points) Danielle buys an Inland Marine policy with a disappearing deductible from Tan Casualty Company. The policy pays zero if the loss is less than 100,000 and pays 100% of the loss if the loss is 900,000 or greater. Danielle has a loss and receives a payment of 560,000 from Tan.

   Determine the amount of the loss.

   Solution:

$$L - \frac{900,000 - L}{900,000 - 100,000}(100,000) = 560,000$$

$$L - \frac{900,000 - L}{8} = 560,000$$

$$8L - 900,000 + L = (8)(560,000)$$

$$9L = 5,380,000$$

$$L = 597,777.78$$

9. (8 points) Stavan purchases a homeowners policy with an 80% coinsurance provision on his house in Florida. The home is insured for 400,000. The home was worth 410,000 on the day the policy was purchased. There is a hurricane that causes 250,000 worth of damage. On the date of the hurricane, the house was worth 550,000.

   Calculate the benefit payment that Stavan will receive under his policy.

   Solution:

$$Payment = Min\left[400{,}000; \frac{400{,}000}{(0.8)(550{,}000)}(250{,}000)\right]$$

$$= 227{,}272.73$$

10. (4 points) List four reasons for deductibles.

    Solution:

    Eliminates small claims which are costly (expense wise) to pay

    Gives the insured options – larger deductibles result in lower premiums and vice versa

    Deductibles incent the insured to avoid losses so it aligns the incentives of the insured with those of the insurance company

    Reduces the premiums overall as the claims paid are reduced by the deductible.