

STAT 479

Quiz 1

January 19, 2012

1. Wang Dental Insurance Company has received the following sample of claims:

80 80 100 80 120

Wang believes that dental claims are distributed as a Gamma distribution with $\alpha = 33$ and $\theta = 2.8$.

VAR SAM is the variance of the sample of claims.

VARDIST is the variance based on the expected distribution.

Calculate VAR SAM - VARDIST.

Sample Data

$$E(X) = \frac{80 + 80 + 100 + 80 + 120}{5} = 92$$

$$E(X^2) = \frac{3(80)^2 + (100)^2 + (120)^2}{5} = 8720$$

$$\text{VAR SAM} = 8720 - (92)^2 = 256$$

Gamma

$$\text{Var} = \alpha \theta^2 = (33)(2.8)^2 = 258.72$$

$$\begin{aligned} \text{VAR SAM} - \text{VARDIST} &= 256 - 258.72 \\ &= -2.72 \end{aligned}$$

2. You are given $F(x) = \frac{x^2}{100}$ for $0 < x < 10$.

$$f(x) = F'(x)$$

Calculate $e(5)$. This is $E[(X-5|X>5)]$.

$$= \frac{2x}{100} = \frac{x}{50}$$

$$\begin{aligned} e(5) &= E[(X-5|X>5)] \\ &= \frac{\int_5^{10} (x-5) f(x) dx}{1 - F(5)} \\ &= \frac{\int_5^{10} (x-5) \left(\frac{x}{50}\right) dx}{1 - \frac{25}{100}} \\ &= \frac{\frac{1}{50} \left(\frac{x^3}{3} - \frac{5x^2}{2} \right) \Big|_5^{10}}{0.75} \end{aligned}$$

$$= \frac{\frac{1}{50} \left[\frac{1000}{3} - \frac{5(100)}{2} \right] - \frac{1}{50} \left[\frac{125}{3} - \frac{125}{2} \right]}{0.75}$$

$$= 2.778$$