

STAT 479

Quiz 2

Spring 2012

January 26, 2012

1. A two point mixture distribution of  $X$  is comprised of the following two distributions:

$X_1$  a. A uniform distribution where random variable is uniformly distributed between 100 and 500.

$X_2$  b. An empirical distribution with a mean of 100 and a standard deviation of 200.

The weight for the uniform distribution is 0.6 and the weight for the empirical distribution is 0.4.

Calculate the variance of  $X$ .

$$\frac{X_1}{E(X_1)} = \int_{100}^{500} x \left(\frac{1}{400}\right) dx = \frac{(500)^2 - (100)^2}{800} = \frac{500+100}{2} = 300$$

$$E(X_1^2) = \int_{100}^{500} x^2 \left(\frac{1}{400}\right) dx = \frac{(500)^3 - (100)^3}{1200} = 103,333.\bar{3}$$

$$a_1 = 0.6$$

$X_2$

$$E(X_2) = 100$$

$$E(X_2^2) = \text{Var}(X_2) + [E(X_2)]^2 = (200)^2 + (100)^2 = 50,000$$

$$a_2 = 0.4$$

$$E(X) = 0.6 E(X_1) + 0.4 E(X_2) = (0.6)(300) + (0.4)(100) = 220$$

$$E(X^2) = 0.6 E(X_1^2) + 0.4 E(X_2^2) = (0.6)(103,333.\bar{3}) + (0.4)(50,000) = 82,000$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 82,000 - (220)^2 = \underline{\underline{33,600}}$$

2. You are given the following empirical distribution:

1 2 3 4 5 6 7 8 9 10

$k_1$  is the value of  $k$  in the standard deviation principle so that the risk measure under the standard deviation principle is equal to  $VaR_{0.75}(X)$ .

$k_2$  is the value of  $k$  in the standard deviation principle so that the risk measure under the standard deviation principle is equal to  $TVaR_{0.75}(X)$ .

Calculate  $k_2 - k_1$ .

$$E(X) = \frac{1+2+3+4+5+6+7+8+9+10}{10} = 5.5$$

$$E(X^2) = \frac{1+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2}{10} = 38.5$$

$$\text{Var}(X) = 38.5 - (5.5)^2 = 8.25$$

$$\sigma = \sqrt{8.25} = 2.8723$$

standard deviation principle =  $\mu + k\sigma$

$$VaR_{0.75}(X) = 8$$

$$5.5 + k_1(2.8723) = 8 \Rightarrow k_1 = 0.8704$$

$$TVaR_{0.75}(X) = \frac{9+10}{2} = 9.5$$

$$5.5 + k_2(2.8723) = 9.5 \Rightarrow k_2 = 1.3926$$

$$k_2 - k_1 = 1.3926 - 0.8704 = \underline{\underline{0.5222}}$$