STAT 479
Spring 2012
Quiz 3

February 2, 2012

1. For a zero modified Poisson distribution, you are given:

   $p_1^M = 0.20$ and
   $p_2^M = 0.12$

   Calculate $F(2)$.

$$p_k^m = \frac{\lambda}{k} p_{k-1}^m \Rightarrow p_2^m = \frac{\lambda}{2} p_1^m$$

$$.12 = \frac{\lambda}{2}(.2)$$

$$\lambda = \frac{.12}{.10}(2) = 1.2$$

$$p_k^m = (1 - p_0^m)\, p_k^T$$

$$p_1^m \; (1 - p_0^m) \; p_1^T$$

$$0.2 = (1 - p_0^m)\left(\frac{\lambda}{e^\lambda - 1}\right) = (1 - p_0^m)\left(\frac{1.2}{e^{1.2} - 1}\right)$$

$$p_0^m = 1 - \frac{0.2\,(e^{1.2} - 1)}{1.2} = 0.613314$$

$$F(2) = p_0^m + p_1^m + p_2^m = 0.613314 + 0.20 + 0.12 = \underline{0.9333}$$

2. The random variable X is the amount of the loss on an automobile accident and is distributed as a gamma distribution given parameter α^g and with parameter θ^g equal to a constant C.

α^g is distributed as a Pareto distribution with parameters of $\alpha^p = 3$ and $\theta^p = 6$.

Further, you are given that the unconditional $E[X] = 3000$.

Calculate unconditional $Var[X]$.

$$E[X] = E[E[X|\alpha^g]] = E[C\alpha^g] = CE[\alpha^g]$$

$$= C\frac{\theta^p}{\alpha^p - 1} = C\left(\frac{6}{2}\right) = 3C = 3000$$

$$\therefore C = 1000$$

$$Var(X) = E[Var[X|\alpha^g]] + Var[E(X|\alpha^g)]$$

$$= E[C^2\alpha^g] + Var[C\alpha^g]$$

$$= C^2E[\alpha^g] + C^2 Var[\alpha^g]$$

$$= (1000)^2\frac{\theta^p}{\alpha^p - 1} + (1000)^2\frac{(\theta^p)^2\alpha^p}{(\alpha^p - 1)^2(\alpha^p - 2)}$$

$$= (1000)^2\left(\frac{6}{2}\right) + (1000)^2\left(\frac{(6)^2(3)}{(2)(1)}\right)$$

$$= \underline{\underline{30{,}000{,}000}}$$