STAT 479
Spring 2012
Quiz 4
February 23, 2012

1. Wang Insurance Company sells a health insurance policy which covers the cost of doctor visits during a calendar year. The number of doctor visits in a year is distributed as geometric distribution with a mean of 10. The cost of each doctor visit is distributed as a Pareto distribution with $\theta = 200$ and $\alpha = 5$.

   Wang charges a premium equal to the expected value of payments plus one standard deviation.

   Calculate the premium.

$$E[N] = 10 = \beta$$

$$Var[N] = (\beta)(1+\beta) = (10)(11) = 110$$

$$E[X] = \frac{\theta}{\alpha - 1} = \frac{200}{5-1} = 50$$

$$Var[X] = \frac{\theta^2 \alpha}{(\alpha-1)^2(\alpha-2)} = \frac{(200)^2(5)}{(4)^2(3)} = \frac{12500}{3}$$

$$E[S] = E[N] \cdot E[X] = (10)(50) = 500$$

$$Var[S] = (E[X])^2 Var[N] + Var[X] E[N]$$

$$= (50)^2 (110) + \left(\frac{12500}{3}\right)(10)$$

$$= 316,666.\overline{6}$$

$$\text{Premium} = 500 + \sqrt{316,666.67} = \underline{1062.73}$$

2. The Boiler Up Game Show (BUGS) can have zero, one, or two winners each show. The following table lists the associated probability for the number of winners.

Number of Winners	Probability
0	10%
1	40%
2	50%

Each winner can win one of three prizes. The value of the prizes and the associated probabilities are listed in the following table:

Value of Prize	Probability
100	20%
300	70%
1000	10%

Crosby Casualty Insurance Company provides BUGS with an insurance policy that pays if the total aggregate prizes in any show exceeds 400. The policy will pay the entire cost of prizes in excess of 400.

Calculate Crosby's expected payout under this policy for each show.

$E[N] = (0.4)(1) + (0.5)(2) = 1.4$

$E[X] = (100)(.2) + (300)(.7) + 1000(.1) = 330$

$E[S] = (1.4)(330) = 462$

$Pr(S=0) = 0.10$

$Pr(S=100) = Pr(N=1) \cdot Pr(X=100) = (0.40)(0.20) = 0.08$

$Pr(S=200) = Pr(N=2) \cdot Pr(X=100) \cdot Pr(X=100) = (.5)(.2)^2 = 0.02$

$Pr(S=300) = Pr(N=1)\ Pr(X=300) = (0.4)(0.7) = 0.28$

$E(S \wedge 400) = (0)(0.1) + (100)(0.08) + 200(0.02) + 300(.28) + 400(1 - 0.1 - 0.08 - 0.02 - 0.28)$

$= 304$

Expected Payout $= E[S] - E[S \wedge 400] = 462 - 304 = \underline{158}$