STAT 479
Spring 2012
Quiz 5
March 1, 2012

1. Students taking Exam C can be categorized into two categories – Well Prepared and Unprepared. The probability of a Well Prepared student passing Exam C is 75%. The probability of an Unprepared student passing the Exam C is 10%.

   Well prepared students who pass the exam receive an exam passing bonus which is distributed as a gamma distribution with $\alpha = 3$ and $\theta = 1000$.

   Unprepared students who pass the exam receive an exam passing bonus that is uniformly distributed between 0 and 4000.

   For this sitting of Exam C, there are 1200 Well Prepared students and 800 Unprepared students.

   Calculate the variance of the exam passing bonus.

WELL PREPARED

$\mu_W = \alpha\theta = (3)(1000) = 3000$

$Var_W = \alpha\theta^2 = 3(1000)^2 = 3,000,000$

UNPREPARED

$\mu_U = \frac{4000}{2} = 2000$

$Var_U = \frac{(4000)^2}{12} = 1,333,333.3\overline{3}$

$$Var(S) = \sum \left[q_i \sigma_i^2 + q_i(1-q_i)\mu_i^2 \right]$$

$$= 1200\left[0.75(3,000,000) + (.75)(.25)(3000)^2\right]$$

$$+ 800\left[0.1(1,333,333.\overline{33}) + (0.1)(0.9)(2000)^2\right]$$

$$= 5,119,666,667$$

2. The number of losses under an insurance policy are distributed as a negative binomial with a mean of 8 and a variance of 40.

The amount of a loss is distributed as a Burr distribution with parameters of α, $\theta = 1000$, and $\gamma = 2$.

A deductible of 500 is implemented. N is the random variable representing the number of claim payments with the deductible. The variance of N is 24.

Determine α.

Negative Binomial

$r\beta = 8 \quad r(\beta)(1+\beta) = 40 \Rightarrow \beta = 4 \text{ \& } r = 2$

$N^P \sim$ Negative Binomial with r and $v\beta$

so 2 and $v \cdot 4$

$$Var(N^P) = r(v\beta)(1+v\beta) = 24$$

$$\Rightarrow 2(v \cdot 4)(1 + v \cdot 4) = 24$$

$$\Rightarrow 32v^2 + 8v - 24 = 0$$

$$\Rightarrow 4v^2 + v - 3 = 0$$

$$\Rightarrow (4v - 3)(v + 1) = 0$$

$$\Rightarrow v = 3/4 \text{ or } \underbrace{v = -1}_{\text{Not possible}}$$

$$v = Pr(X > 500) = 1 - F(500) = 1 - \left[1 - \left(\frac{1}{1 + \frac{500}{1000}}\right)^{\alpha}\right]$$

$$3/4 = \left(\frac{1}{1.25}\right)^{\alpha} \qquad \alpha = \frac{\ln(3/4)}{\ln(.8)} = \underline{1.2892}$$