STAT 479

Test 1

Spring 2012

February 14, 2012

1. Losses during 2011 were distributed as a gamma distribution with a variance of 4,480,000.

   Losses are expected to be subject to P % inflation for 2012. The variance of losses for 2012 is expected to be 17,920,000.

   Determine P.

Var of a gamma distribution $= \alpha\theta^2$

2011 $\alpha\theta_{2011}^2 = 4,480,000$

2012 $\theta_{2012} = (1 + P/100)\,\theta_{2011}$ because θ_{2011} is a scale parameter

$\text{Var}_{2012} = \alpha(\theta_{2012})^2 = \alpha(1 + P/100)^2(\theta_{2011})^2$

$17,920,000 = (1 + P/100)^2(4,480,000)$

$4 = (1 + P/100)^2$

$2 = 1 + P/100 \quad \therefore P = 100\%$

2. The cost of one day in the hospital is distributed as a two point mixture distribution. The distribution is 0.7 of a Pareto distribution with parameters of $\theta = 5000$ and $\alpha = 5$ and 0.3 of a Gamma distribution with a mean of 10,000 and a variance of 5 million.

   Hassan Hospital has 100 patients today. Assume the cost of each patient is independent of the cost of any other patient.

   Using the normal approximation, calculate the probability that their total cost will exceed 500,000.

$$E(Y) = 0.7\left(\frac{5000}{4}\right) + 0.3(10000) = 3875$$

$$E(Y^2) = 0.7\left(\frac{(2)(5000^2)}{(4)(3)}\right) + 0.3\left(\underbrace{5{,}000{,}000}_{Var} + \underbrace{10{,}000^2}_{(E(X))^2}\right)$$

$$= 34{,}416{,}666.6\bar{6}$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 19{,}401{,}041.6\bar{6}$$

$$E(S) = 100(3875) = 387{,}500$$

$$Var(S) = 100(19{,}401{,}041.6\bar{6}) = 1{,}940{,}104{,}166.\bar{6}$$

$$Pr(S > 500{,}000) = P\left(Z > \frac{500{,}000 - 387{,}500}{\sqrt{1{,}940{,}104{,}166.\bar{6}}}\right)$$

$$= Pr(Z > 2.55)$$

$$= 1 - 0.9946 = \underline{0.0054}$$

3. The number of vehicles that arrive at a gas station during an hour is distributed as a Poisson distribution with a mean of 60. Of every 500 vehicles that arrive at the gas station, one is a Toyota Prius.

Calculate the probability that the number of Priuses that arrive at a gas station in a 24 hour period will exceed the expected number of Priuses.

E(Number of PRIUSES in 24 hours)

$$= 60 \times 24 \times \frac{1}{500} = 2.8$$

Pr (Number > 2.8) =

$$1 - Pr(0) - Pr(1) - P(2)$$

$$= 1 - e^{-2.8} - \frac{2.8e^{-2.8}}{1} - \frac{(2.8)^2 e^{-2.8}}{2}$$

$$= 1 - e^{-2.8}\left[1 + 2.8 + \frac{(2.8)^2}{2}\right]$$

$$= \underline{\underline{0.5494}}$$

4. The number of dropped calls in a month on an iPhone is distributed as a Negative Binomial with a mean of 8 and a variance of 40.

N represents the number of dropped calls for any other cell phone which is distributed as a Negative Binomial with the same parameters as the iPhone except that the probability of having no dropped calls in a month is set to 20%.

Calculate the $Var[N]$.

IPHONES

$$E(N) = \gamma\beta = 8 \qquad Var(N) = \gamma\beta(1+\beta) = 40$$

$$\frac{\gamma(\beta)(1+\beta)}{\gamma\beta} = 1+\beta = \frac{40}{8} = 5 \Rightarrow \beta = 4$$

$$\gamma = 2$$

Other phones

Zero modified Negative Binomial

$$p_0^m = 0.20 \qquad \beta = 4 \quad \gamma = 2$$

$$Var(N) = (1-p_0^m)(\text{zero truncated Var}) + (p_0^m)(1-p_0^m)(\text{zero truncated mean})^2$$

$$\text{zero truncated variance} = \frac{\gamma\beta\left[(1+\beta) - (1+\beta+\gamma\beta)(1+\beta)^{-\gamma}\right]}{\left[1-(1+\beta)^{-\gamma}\right]^2}$$

$$= \frac{(2)(4)\left[5 - 13(5)^{-2}\right]}{\left(1-(5)^{-2}\right)^2} = 38.\overline{88}$$

$$\text{Zero truncated mean} = \frac{\gamma\beta}{1-(1+\beta)^{-\gamma}} = \frac{(2)(4)}{1-(5)^{-2}} = 8.3\overline{3}$$

$$Var(N) = 0.8\,(38.\overline{88}) + (.2)(.8)(8.3\overline{3})^2 = \underline{42.2\overline{2}}$$

5. The number of claims associated with a critical illness policy, represented by the random variable N, is distributed as a Poisson distribution with a mean of λ. Further, you are given that λ is distributed as an exponential distribution with a mean of 1.3.

Calculate the $Var[N]$.

$$Var[N] = E(Var(N|\lambda)) + Var[E(N|\lambda)]$$

$$= E[\lambda] + Var[\lambda]$$

$$= 1.3 + (1.3)^2$$

$$= 2.99$$

Exponential $\theta = 1.3$

$E(\lambda) = \theta = 1.3$

$Var(\lambda) = \theta^2 = (1.3)^2$

6. In 2011, losses were uniformly distributed between 0 and 50,000. An insurance policy pays for all claims in excess of an **franchise** deductible of $5,000. For 2012, claims will be subject to uniform inflation of 8%. The insurance company replaces the franchise deductible with an **ordinary** deductible of d so that the expected cost per loss in 2011 is the same as the expected cost per loss in 2012. Calculate d.

2011

$$E[Y^L] = \int_{5000}^{50000} \frac{50{,}000 - x}{50000}\,dx + 5000\,(1 - F(5000))$$

$$= x - \frac{x^2}{100{,}000}\Big|_{5000}^{50000} + 5000(.9)$$

$$= 24{,}750$$

OR

$$E[Y^L] = \int_{5000}^{50000} x\left(\frac{1}{50000}\right)dx = \frac{x^2}{100{,}000}\Big|_{5000}^{50000}$$

$$= 24{,}750$$

OR $E[X] - E[X \wedge 5000] + 5000\,[1 - F(5000)]$

$$= \frac{50000}{2} - \left[\frac{1}{10}\left(\frac{5000}{2}\right) + \frac{9}{10}(5000)\right] + 5000(.9)$$

$$= 24{,}750$$

For 2012

$$24{,}750 = 1.08\left[E(X) - E\left(X \wedge \frac{d}{1.08}\right)\right]$$

$$= 1.08 \int_{\frac{d}{1.08}}^{50{,}000} \frac{50000 - x}{50000}\,dx$$

$$\frac{24{,}750}{1.08} = x - \frac{x^2}{100{,}000}\Big|_{\frac{d}{1.08}}^{50000} = 50{,}000 - \frac{50000^2}{100000} - \frac{d}{1.08} + \frac{d^2}{116{,}640}$$

$$\frac{d^2}{116{,}640} - \frac{d}{1.08} + 2083.3\overline{3} = 0$$

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① $d^2 - 108{,}000\,d + 242{,}999{,}999.6 = 0$

$$d = \frac{108{,}000 \pm \sqrt{(108{,}000)^2 - 4(242{,}999{,}999.6)}}{2}$$

$= 2298.94$

OR We can redefine the range to 0 - 54,000

since $\theta(1.08) = 54{,}000$

$$24{,}750 = \int_d^{54000} \frac{54{,}000 - x}{54000}\,dx = x - \frac{x^2}{108{,}000}\Big|_d^{54000}$$

$$54{,}000 - \frac{(54000)^2}{108{,}000} - d + \frac{d^2}{108{,}000} = 24750$$

$$\frac{d^2}{108{,}000} - d + 2250 = 0$$

$d^2 - 108{,}000\,d + 243{,}000{,}000 = 0$ SAME as ①

$d = 2298.94$

OR $E[X] - E[X \wedge d] = 24{,}750$

$$\frac{54000}{2} - \frac{d}{54000}\left(\frac{d}{2}\right) - d\left(\frac{54000-d}{54000}\right) = 24{,}750$$

$\Rightarrow d^2 - 108{,}000\,d + 243{,}000{,}000 = 0$

$d = 2298.94$

OR $E[(X-d)_+] = \dfrac{54000-d}{54000} \cdot \dfrac{54000-d}{2} = 24{,}750$

Prob of exceed deductible

Ave amount when deductible is exceed

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$$(54000 - d)^2 = (24{,}750)(2)(54000)$$

$$54000 - d = \sqrt{2{,}673{,}000{,}000} = 51701.06$$

$$d = \underline{\underline{2298.94}}$$

7. Losses represented by X are distributed as a Pareto distribution with parameters $\theta = 60$ and $\alpha = 2$.

$$R_X = \frac{TVaR_{.75}(X)}{VaR_{.75}(X)}.$$

Losses represented by Y are distributed uniformly between 0 and 100.

$$R_Y = \frac{TVaR_p(Y)}{VaR_p(Y)}.$$

Determine p such that $R_X = R_Y$.

Pareto

$$VaR_{0.75}(x) = \theta\left[(1-p)^{-\frac{1}{\alpha}} - 1\right] = 60\left[.25^{-1/2} - 1\right] = 60$$

$$TVaR_{0.75}(x) = VaR_{0.75}(x) + \frac{\theta[1-p]^{-1/\alpha}}{\alpha - 1}$$

$$= 60 + \frac{60(1-.75)^{-1/2}}{1} = 180$$

$$R_X = \frac{180}{60} = 3$$

Uniform

Derived in Homework

$$VaR_p = a(1-p) + p(b) = (0)(1-p) + p(100) = 100p$$

$$TVaR_p = \frac{a(1-p) + pb + b}{2} = \frac{0(1-p) + p(100) + 100}{2}$$

$$= 50(1+p)$$

$$R_Y = 3 = \frac{50(1+p)}{100p} \Rightarrow 300p = 50 + 50p$$

$$250p = 50$$

$$p = \frac{50}{250} = \underline{\underline{0.2}}$$

8. You are given the following losses suffered by policyholders of Datsenka Dental Insurance Company:

45, 50, 50, 50, 60, 75, 80, 120, 230

The random variable X represents the losses incurred by the policyholders. Treating this data as an empirical distribution, calculate:

a. $E[X]$
b. $Var[X]$
c. The mode of the distribution
d. Datsenka has issued a dental policy that has an ordinary deductible of 50 and an upper limit of 100. The upper limit is applied to the loss before the deductible. Calculate Datsenka's expected payment per payment.

(a) $E[X] = \dfrac{45+50+50+50+60+75+80+120+230}{9} = \dfrac{760}{9} = 84.\overline{44}$

(b) $E[X^2] = \dfrac{45^2+(50^2)3+60^2+75^2+80^2+120^2+130^2}{9}$

$= \dfrac{92450}{9} = 10,272.2\overline{2}$

$Var[X] = \dfrac{92450}{9} - \left(\dfrac{760}{9}\right)^2 = 3141.358$

(c) Mode = Most commm value = 50

(d) $\dfrac{E[X \wedge 100] - E[X \wedge 50]}{1 - F[50]} = \dfrac{67.\overline{77} - 49.\overline{44}}{1 - 4/9} = 33$

$E[X \wedge 100] = \dfrac{45 + (3)(50) + 60 + 75 + 80 + 100 + 100}{9} = \dfrac{610}{9} = 67.\overline{77}$

$E[X \wedge 50] = \dfrac{45 + (8)(50)}{9} = \dfrac{445}{9} = 49.\overline{44}$ $\qquad F[50] = 4/9$

we can calculate directly as $\dfrac{10 + 25 + 30 + 50 + 50}{5} = 33$

$60 \to 10$
$75 \to 25$
$80 \to 30$
$120 \to 50$
$230 \to 50$

9. You are given the following empirical distribution of losses:

1000 4000 5000 6000 10,000

Wang Insurance Company wants to apply an ordinary deductible to these claims so that the Loss Elimination Ratio will be 0.30.

Determine the deductible.

$$E[X] = \frac{1000 + 4000 + 5000 + 6000 + 10{,}000}{5} = 5200$$

$$LER = 0.3 = \frac{E[X \wedge d]}{E[X]} = \frac{E[X \wedge d]}{5200}$$

$$E[X \wedge d] = 5200(0.3) = 1560$$

$$\therefore \frac{\Sigma (x \wedge d)}{5} = 1560 \Rightarrow \Sigma(x \wedge d) = (1560)(5) = 7800$$

$d < 1000$ means that $\Sigma(x \wedge d)$ must be less than 5000 so d cannot be less than 1000

$1000 < d < 4000$ means 1000 pd for 1000 claim and d for others

so $1000 + 4d = 7800$

$4d = 6800$

$\underline{d = 1700}$ no contradiction.

10. You are given $f(x)=\frac{3x^2}{1000}$ for $0 \le x \le 10$. Calculate the median of this distribution.

$$F(x) = \int_0^x \frac{3t^2}{1000}\,dt = \frac{t^3}{1000}\Big|_0^x = \frac{x^3}{1000}$$

$$\text{Median} = 0.5 = F(\pi_{0.5}) = \frac{\pi_{0.5}^3}{1000}$$

$$500 = (\pi_{0.5})^3$$

$$\pi_{0.5} = \text{Median} = (500)^{1/3} = \underline{\underline{7.937}}$$