STAT 479
Test 3
Fall 2014

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The Wang Warranty Company provides warranty coverage for cars manufactured by Amstutz Automobile Company whose President is Kevin. The coverage provided to Amstutz does not include any deductible or upper limits. Wang wants to model the number of claims experienced by each warranty. They hire the consulting firm of Zhang & Zhang.

Bolun, one of the partners of Zhang & Zhang, believes that the number of claims will be distributed as a Poisson distribution. He selects a sample of 10 warranties and determines the number of claims for those 10 warranties. The results of the sample was the following number of claims per warranty.

Number of Claims	Warranties
0	3
1	4
2	2
3	1

This data will be used for Questions 1 and 2.

Bolun wants to provide a linear confidence interval for the parameter λ. He uses the Maximum Likelihood Estimate for λ and produces a 90% confidence interval for λ.

1. (5 points) What was his confidence interval?

Shaotian, the other partner in Zhang & Zhang, believes that the number of claims is better modeled using a binomial distribution. Shaotian uses the Method of Moments to estimate the parameters to use with the binomial distribution.

The sample had the following number of claims per warranty.

Number of Claims	Warranties
0	3
1	4
2	2
3	1

This data is copied from above for your convenience and is used in Questions 1 and 2.

2. (7 points) Calculate the parameters estimated by Shaotian.

Minkang, one the owners of Wang Warranty, decides that the Company can produce better data for the number of claims and develops the following table of number of claims by warranty:

Number of Claims	Warranties
0	3100
1	3600
2	2100
3	900
4 or more	300

Minkang believes that the number of claims is distributed as a Poisson. Minkang has even estimated that λ =1.2. However, he does not remember how to test this hypothesis so he hires Lingxiao from Chen χ^2 Testing to test the following Hypothesis at a 1% significance level:

H_0: The number of claims per warranty is distributed as a Poisson distribution.

H_1: The number of claims per warranty is not distributed as a Poisson distribution.

3. (12 points) Calculate the χ^2 test statistic.

4. (3 points) State the critical value for Lingxiao's test.

5. (2 points) State Lingxiao's conclusion.

Jingkai who is the President of Wang Warranty believes the Company should be concerned with the amount of each claim instead of with the frequency of claims. Jinkai retains Liren who is a classmate from Purdue to study the amount of the claims. Liren selects a sample of claims as follows:

100 500 900

This sample is used for Questions 6-11.

Liren believes that the amount of claims is distributed uniformly between zero and U. Liren wants to use three methods to estimate U. Liren asks Gloria and Yong to help with this project.

6. (3 points) Liren uses the method of moments to estimate U. Determine Liren's estimate of U.

7. (3 points) Gloria uses percentile matching at the 60th percentile to estimate U. Determine Gloria's estimate of U.

8. (4 points) Explain why percentile matching at the 40th percentile would not have been acceptable.

9. (2 points) Yong uses the Maximum Likelihood estimate to estimate U. Determine Yong's estimate of U.

The claim sample selected was claims as follows:

100 500 900

This sample is the same as above and is repeated here for convenience. It is used for Questions 6-11.

Yue, who is the Chief Actuary at Wang, does not believe that the claims are distributed uniformly. She believes that they are distributed as a normal distribution. She asks Antong to develop parameters for the normal distribution using the sample of claims and the Maximum Likelihood Estimator of μ and σ.

10. (6 points) Determine the estimated parameters determined by Antong.

The claim sample selected was claims as follows:

100 500 900

This sample is the same as above and is repeated here for convenience. It is used for Questions 6-11.

Yue also wants to explore the possibility that the claims are distributed as a Gamma distribution. She asks Zichen to develop the parameters for the Gamma distribution using the Method of Moment Matching.

11. (6 points) Determine the estimated parameters determined by Zichen.

Yue decides the sample is too small and decides to expand the sample to five by selecting two additional claims. When she does this, her sample is now:

100 500 500 500 900

Yue is now convinced that the claims are distributed as a normal distribution and wants to test the following Hypothesis:

H_0: The amount of the claim is distributed as a normal distribution with mean of 500 and variance of 80,000.

H_1: The amount of the claim is not distributed as a normal distribution with mean of 500 and variance of 80,000.

Devin is asked to test this hypothesis using the Kolmogorov-Smirnov Test at a significance level of 10%.

12. (12 points) Calculate the test statistic determined by Devin.

13. (2 points) Calculate the critical value for this test.

14. (2 points) State Devin's conclusion regarding Yue's hypothesis.

Wang also provides warranty coverage for Amstutz's chief competitor, Clauss Car Company. The coverage provided to Clauss has no deductible, but there is an upper limit of 2000 for each claim You are given the following sample of claim payments:

500 1000 2000 2000

This data is used for Questions 15-18.

Emily has always like the exponential distribution so she decides to model the claims as an exponential distribution. Being a well trained actuary, she decides to use the Maximum Likelihood Estimator to determine θ.

15. Determine Emily's estimate of θ.

You are given the following sample of claim payments:

500 1000 2000 2000

This data is the same as above and repeated here for your convenience. It is used for Questions 15-18.

Unfortunately, Dora who is an expert in pet insurance but not as knowledgeable about warranty coverage, priced the warranty coverage using an exponential distribution with $\theta = 1000$. Now, Dora wants to test the following hypothesis using the Likelihood Ratio Test at a significance level of 10%:

H_0: The amount of the claims is distributed as an exponential distribution with mean of 1000.

H_1: The amount of the claims is distributed as an exponential distribution.

16. (10 points) Calculate the test statistic for this Likelihood Ratio Test.

17. (2 points) Calculate the critical value.

18. (2 points) State Dora's conclusion.

Wang expanded the sample of data and developed the following data for 20 claims:

Amount of Claim	Number of Claims
0 - 1000	4
1000 - 2000	11
2000 - ∞	5

Yue asks Jack to model these claims using an exponential distribution with $\hat{\theta}$ calculated using the Maximum Likelihood Estimator.

19. (12 points) Determine $\hat{\theta}$.

Wang wants to provide a warranty coverage beginning in 2015 to Cars by Kang Inc. The coverage will have a deductible of 100 for each warranty claim. Further, there will be a maximum that will apply per warranty, not per claim. That maximum will be 1000. In other words, the maximum that Wang can pay out is 1000 on a warranty without regard to the amount or number of claims.

Wang hires Shyu & Spears to simulate claims for this coverage.

The number of claims under each warranty policy is assumed to be distributed as a Poisson distribution with $\lambda = 1.2$.

The amount of each claim under the warranty policy is assumed to be distributed as an Exponential distribution with $\theta = 600$.

Sandy generates the following random numbers:

0.40 0.50 0.92 0.10 0.80 0.50 0.01 0.20

Jacqui uses these random numbers and the inversion method of simulation for each warranty. She is going to manually simulate the amount of claim payments for the first two policies which belong to Marissa and Brandon. First, Jacqui will simulate the number of claims for Marissa and then the amount of claims for Marissa. Then, she will simulate the number of claims for Brandon and then the amount of claims for Brandon.

20. (13 points) Determine the amount of claim payments made for Marissa and for Brandon.

Darmen who is the CFO at Wang Warranty has always been fascinated by the mode of a distribution. During his studies at Purdue, he was an expert on mode, but he has forgotten a lot of what he learned at Purdue because he has not used it recently. However, he decides that he wants to estimate the mode of the distribution of losses on coverage provided to Amstutz Auto. He decides to estimate the mode using:

$$\widehat{Mode} = \overline{X}$$

He collected the following sample of claims:

700 700 1900

Darmen asks Jacob to calculate the Mean Square Error of this estimator using bootstrapping.

21. (9 points) Determine the Mean Square Error.