Question 1 (Orange) or Question 4 (Yellow)

Solution:

\[ E[N^T] = \frac{\gamma \beta}{1 - (1 + \beta)^{-2}} = \frac{(4)(2)}{1 - (1 + 4)^{-2}} = 8.3333 \]

\[ E[N] = (1 - p_0^M)E[N^T] = (1 - 0.6)(8.3333) = 5 \]

\[ Var[N^T] = \frac{\gamma \beta[(1 + \beta) - (1 + \beta + \gamma \beta)(1 + \beta)^{-\gamma}]}{[1 - (1 + \beta)^{-\gamma}]^2} = \]

\[ \frac{(4)(2)[5 - (1 + 4 + 2 \cdot 4)(5^{-3})]}{[1 - 5^{-2}]^2} = 38.8889 \]

\[ Var[N] = (1 - p_0^M)Var[N^T] + (p_0^M)(1 - p_0^M)(E[N^T])^2 \]

\[ = (1 - 0.4)(38.8889) + (0.4)(1 - 0.6)(8.3333)^2 = 40 \]

\[ E[N] + \sqrt{Var[N]} = 5 + \sqrt{40} = 11.3246 \]

Question 2 (Orange) or Question 8 (Yellow)

Solution:

\[ E[X^{Male}] = \alpha \theta = (5)(200) = 1000 \text{ and } Var[X^{Male}] = \alpha \theta^2 = (5)(200^2) = 200,000 \]

\[ E[X^{Female}] = \theta = 600 \text{ and } Var[X^{Female}] = \theta^2 = 600^2 = 360,000 \]

\[ E[S] = (400)(1000) + (200)(600) = 520,000 \]

\[ Var[S] = (400)(200,000) + (200)(360,000) = 152,000,000 \]

\[ Pr(S < 530,000) = Pr\left( Z < \frac{530,000 - 520,000}{\sqrt{152,000,000}} \right) = Pr(Z < 0.8111) = 0.7910 \]
Question 3 (Orange) or Question 7 (Yellow)

Solution:

\[ TVaR_{0.9}(x) = \frac{\int_{x}^{100} x \cdot f(x) \cdot dx}{1 - 0.9} \]

\[ F(\pi_{0.9}) = 0.9 \implies 1 - S(x) \implies 0.9 = \frac{x^2}{10,000} \implies x = \sqrt{9000} = 94.86832981 \]

\[ f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} \left( \frac{x^2}{10,000} \right) = \frac{2x}{10,000} = \frac{x}{5000} \]

\[ TVaR_{0.9}(x) = \frac{\int_{94.86832981}^{100} x \cdot \frac{x}{5000} \cdot dx}{1 - 0.9} = \frac{\int_{94.86832981}^{100} \frac{x^2}{5000} \cdot dx}{0.1} = 10 \left[ \frac{x^3}{15,000} \right]_{94.86832981}^{100} = 100^3 - 94.86832981^3 \]

\[ \frac{1500}{1500} = 97.4567 \]
Question 4 (Orange) or Question 2 (Yellow)

Solution

\[ \text{Var}[X] = \text{Var}[E[X | B]] + E[\text{Var}[X | B]] \]

\[ E[X | B] = \frac{B}{2} \quad \text{and} \quad \text{Var}[X | B] = \frac{B^2}{12} \]

\[ \text{Var}[X] = \text{Var} \left( \frac{B}{2} \right) + E \left( \frac{B^2}{12} \right) = \frac{1}{4} \text{Var}[B] + \frac{1}{12} E[B^2] \]

\[ \text{Var}[B] = \frac{\alpha \theta^2}{(\alpha - 1)^2(\alpha - 2)} = \frac{(3)(400^2)}{(2)^2(1)} = \frac{120,000}{1} \]

\[ E[B^2] = \frac{\theta^2(2!)}{(\alpha - 1)(\alpha - 2)} = \frac{400^2}{2(1)} = 160,000 \]

\[ \text{Var}[X] = \frac{1}{4} (120,000) + \frac{1}{12} (160,000) = \frac{130,000}{3} = 43,333.33 \]
Question 5 – Part 1

Solution:

\[ E[X \wedge u] = 0.8E[X] \]

\[
\frac{\theta}{\alpha-1} \left[ 1 - \left( \frac{\theta}{\theta + u} \right)^{\alpha-1} \right] = 0.8 \left( \frac{\theta}{\alpha-1} \right)
\]

\[
\frac{100,000}{3-1} \left[ 1 - \left( \frac{100,000}{100,000 + u} \right)^{3-1} \right] = 0.8 \left( \frac{100,000}{3-1} \right) = 40,000
\]

\[
1 - \left( \frac{100,000}{100,000 + u} \right)^2 = \frac{40,000}{50,000} = 0.8 \implies \frac{100,000}{100,000 + u} = \sqrt{0.2}
\]

\[
u = \frac{100,000}{\sqrt{0.2}} - 100,000 = 123,606.80
\]

Question 5 – Part 2

Solution:

\[ X^{2016} \sim \text{Pareto with } \theta = 110,000 \text{ and } \alpha = 3 \]

\[ E[X^{2016}] - E[X^{2016} \wedge d] = 0.8E[X^{2015}] = 40,000 \]

\[
\left( \frac{\theta^{2016}}{\alpha-1} \right) \left( \frac{\theta^{2016}}{\theta^{2016} + d} \right)^{\alpha-1} = 40,000
\]

\[
\left( \frac{110,000}{2} \right) \left( \frac{110,000}{110,000 + d} \right)^2 = 40,000
\]

\[ \implies d = 110,000 \left( \frac{55,000}{40,000} \right)^{110,000} = 18,986.43 \]
Question 6 (Orange) or Question 1 (Yellow)

Solution:
The number of qualified candidates in a 5 day period is \((3)(5)(0.12) = 1.8 = \lambda_{\text{Qualified}}\)

Probability of at least two qualified candidates = \(1 - p_0 - p_1\)

\[= 1 - e^{-1.8} - (1.8)e^{-1.8} = 0.5372\]

Question 7 (Orange) or Question 3 (Yellow)

Solution:

\[\text{Bonus} = 0.2[24,000 - E[X \wedge 24,000]] - 0.2[E[X] - E[X \wedge 48,000]]\]

\[= 0.2[24,000 - 24,000(1 - e^{-1})] - 0.2[24,000 - 24,000(1 - e^{-2})]\]

\[= 1765.82 - 649.61 = 1116.21\]

Question 8 (Orange) or Question 2 (Yellow)

Solution:

\[E[X] = 0.6E[X_1] + 0.4E[X_2] = (0.6)\left(\frac{1000}{4}\right) + (0.4)\left(\frac{6000}{2}\right) = 1350\]

\[E[X^2] = 0.6E[X_1^2] + 0.4E[X_2^2] =\]

\[= (0.6)\left(\frac{2\cdot1000^2}{4\cdot3}\right) + (0.4)\left(\frac{2\cdot6000^2}{2\cdot1}\right) = 14,500,000\]

\[Var[X] = E[X^2] - (E[X])^2 = 14,500,000 - (1350)^2 =\]

\[12,677,500\]
Question 9

Solution:

<table>
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<tr>
<th>x</th>
<th>f(x)</th>
<th>Y^p</th>
<th>f(y)</th>
</tr>
</thead>
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<tr>
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<td>0.2</td>
<td>0</td>
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</tr>
<tr>
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<td>25</td>
<td>0.25</td>
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<td>105</td>
<td>0.25</td>
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<tr>
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<td>0.2</td>
<td>275</td>
<td>0.25</td>
</tr>
<tr>
<td>1000</td>
<td>0.2</td>
<td>675</td>
<td>0.25</td>
</tr>
</tbody>
</table>

\[ E[Y^p] = (0.25)(25) + (0.25)(105) + (0.25)(275) + (0.25)(675) = 270 \]

\[ Var[Y^p] = E[(Y^p)^2] - (E[Y^p])^2 \]

\[ E[(Y^p)^2] = (0.25)(25^2) + (0.25)(105^2) + (0.25)(275^2) + (0.25)(675^2) = 135,725 \]

\[ Var[Y^p] = 135,725 - (270)^2 = 62,825 \]