Question 1 (Orange) or Question 4 (Yellow)

Solution:

$$E[N^T] = \frac{\gamma\beta}{1-(1+\beta)^{-2}} = \frac{(4)(2)}{1-(1+4)^{-2}} = 8.3333$$

$$E[N] = (1-p_0^M)E[N^T] = (1-0.6)(8.3333) = 5$$

$$Var[N^T] = \frac{\gamma\beta[(1+\beta)-(1+\beta+\gamma\beta)(1+\beta)^{-\gamma}]}{[1-(1+\beta)^{-\gamma}]^2} =$$

$$\frac{(4)(2)[5-(1+4+2\cdot 4)(5^{-2})}{[1-5^{-2}]^2} = 38.88889$$

$$Var[N] = (1-p_0^M)Var[N^T] + (p_0^M)(1-p_0^M)\left(E[N^T]\right)^2$$

$$= (1-0.4)(38.88889) + (0.4)(1-0.6)(8.33333)^2 = 40$$

$$E[N] + \sqrt{Var[N]} = 5 + \sqrt{40} = 11.3246$$

Question 2 (Orange) or Question 8 (Yellow)

Solution:

$$E[X^{Male}] = \alpha\theta = (5)(200) = 1000 \text{ and } Var[X^{Male}] = \alpha\theta^2 = (5)(200^2) = 200,000$$

$$E[X^{Female}] = \theta = 600 \text{ and } Var[X^{Female}] = \theta^2 = 600^2 = 360,000$$

$$E[S] = (400)(1000) + (200)(600) = 520,000$$

$$Var[S] = (400)(200,000) + (200)(360,000) = 152,000,000$$

$$\Pr(S < 530,000) = \Pr\left(Z < \frac{530,000-520,000}{\sqrt{152,000,000}}\right) = \Pr(Z < 0.8111) = 0.7910$$

Question 3 (Orange) or Question 7 (Yellow)

Solution:

$$TVaR_{0.9}(x) = \frac{\int_{\pi_{0.9}}^{100} x \cdot f(x) \cdot dx}{1-0.9}$$

$$F(\pi_{0.9}) = 0.9 ==> 1 - S(x) ==> 0.9 = \frac{x^2}{10,000} = 0.9 ==> x = \sqrt{9000} = 94.86832981$$

$$f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{x^2}{10,000}\right) = \frac{2x}{10,000} = \frac{x}{5000}$$

$$TVaR_{0.9}(x) = \frac{\int_{94.86832981}^{100} x \cdot \frac{x}{5000} \cdot dx}{1-0.9} = \frac{\int_{94.86832981}^{100} \frac{x^2}{5000} \cdot dx}{0.1} = 10\left[\frac{x^3}{15,000}\right]_{94.86832981}^{100} =$$

$$\frac{100^3 - 94.86832981^3}{1500} = 97.4567$$

Question 4 (Orange) or Question 2 (Yellow)

Solution

$$Var[X] = Var[E[X \mid B]] + E[Var[X \mid B]]$$

$$E[X \mid B] = \frac{B}{2} \text{ and } Var[X \mid B] = \frac{B^2}{12}$$

$$Var[X] = Var\left[\frac{B}{2}\right] + E\left[\frac{B^2}{12}\right] = \frac{1}{4}Var[B] + \frac{1}{12}E[B^2]$$

$$Var[B] = \frac{\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = \frac{(3)(400^2)}{(2)^2(1)} = 120,000$$

$$E[B^2] = \frac{\theta^2(2!)}{(\alpha-1)(\alpha-2)} = \frac{400^2(2)}{(2)(1)} = 160,000$$

$$Var[X] == \frac{1}{4}(120,000) + \frac{1}{12}(160,000) = \frac{130,000}{3} = 43,333.33$$

Question 5 – Part 1

Solution:

$$E[X \wedge u] = 0.8E[X]$$

$$\frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{\theta+u}\right)^{\alpha-1}\right] = 0.8\left(\frac{\theta}{\alpha-1}\right)$$

$$\frac{100,000}{3-1}\left[1-\left(\frac{100,000}{100,000+u}\right)^{3-1}\right] = 0.8\left(\frac{100,000}{3-1}\right) = 40,000$$

$$1-\left(\frac{100,000}{100,000+u}\right)^{2} = \frac{40,000}{50,000} = 0.8 ==> \frac{100,000}{100,000+u} = \sqrt{0.2}$$

$$u = \frac{100,000}{\sqrt{0.2}} - 100,000 = 123,606.80$$

Question 5 – Part 2

Solution:

$$X^{2016} \sim Pareto\ with\ \theta = 110,000\ and\ \alpha = 3$$

$$E[X^{2016}] - E[X^{2016} \wedge d] = 0.8E[X^{2015}] = 40,000$$

$$\left(\frac{\theta^{2016}}{\alpha-1}\right)\left(\frac{\theta^{2016}}{\theta^{2016}+d}\right)^{\alpha-1} = 40,000$$

$$\left(\frac{110,000}{2}\right)\left(\frac{110,000}{110,000+d}\right)^{2} = 40,000$$

$$==> d = 110,000\left(\sqrt{\frac{55,000}{40,000}}\right) - 110,000 = 18,986.43$$

Question 6 (Orange) or Question 1 (Yellow)

Solution:

The number of qualified candidates in a 5 day period is $(3)(5)(0.12) = 1.8 = \lambda^{Qualified}$

Probability of at least two qualified candidates = $1 - p_0 - p_1$

$$= 1 - e^{-1.8} - (1.8)e^{-1.8} = 0.5372$$

Question 7 (Orange) or Question 3 (Yellow)

Solution:

$$Bonus = 0.2\{24,000 - E[X \wedge 24,000]\} - 0.2\{E[X] - E[X \wedge 48,000]\}$$

$$= 0.2\{24,000 - 24,000(1 - e^{-1})\} - 0.2\{24,000 - 24,000(1 - e^{-2})\}$$

$$= 1765.82 - 649.61 = 1116.21$$

Question 8 (Orange) or Question 2 (Yellow)

Solution:

$$E[X] = 0.6E[X_1] + 0.4E[X_2] = (0.6)\left(\frac{1000}{4}\right) + (0.4)\left(\frac{6000}{2}\right) = 1350$$

$$E[X^2] = 0.6E[X_1^2] + 0.4E[X_2^2] =$$

$$(0.6)\left(\frac{2 \cdot 1000^2}{4 \cdot 3}\right) + (0.4)\left(\frac{2 \cdot 6000^2}{2 \cdot 1}\right) = 14,500,000$$

$$Var[X] = E[X^2] - (E[X])^2 = 14,500,000 - (1350)^2 =$$

$$12,677,500$$

Question 9

Solution:

x	f(x)	Y^P	f(y)
100	0.2	0	Undefined
150	0.2	25	0.25
230	0.2	105	0.25
400	0.2	275	0.25
1000	0.2	675	0.25

$$E[Y^P] = (0.25)(25) + (0.25)(105) + (0.25)(275) + (0.25)(675) = 270$$

$$Var[Y^p] = E[(Y^P)^2] - (E[Y^P])^2$$

$$E[(Y^P)^2] = (0.25)(25^2) + (0.25)(105^2) + (0.25)(275^2) + (0.25)(675^2) = 135,725$$

$$Var[Y^p] = 135,725 - (270)^2 = 62,825$$