

## Homework Problems

### Stat 479

#### Chapter 2

1. Model 1 in the table handed out in class is a uniform distribution from 0 to 100. Determine what the table entries would be for a generalized uniform distribution covering the range from  $a$  to  $b$  where  $a < b$ .
2. Let  $X$  be a discrete random variable with probability function  $p(x) = 2(1/3)^x$  for  $x = 1, 2, 3, \dots$ . What is the probability that  $X$  is odd?
3. \* For a distribution where  $x \geq 2$ , you are given:
  - The hazard rate function:  $h(x) = z^2/2x$ , for  $x \geq 2$
  - $F(5) = 0.84$ .

Calculate  $z$ .

4.  $F_X(t) = (t^2-1)/9999$  for  $1 < t < 100$ . Calculate  $f_X(50)$ .
5. You are given that the random variable  $X$  is distributed as a Weibull distribution with parameters  $\theta = 3$  and  $\tau = 0.5$ . Calculate:
  - a.  $\Pr[X \leq 5]$
  - b.  $\Pr[3 \leq X \leq 5]$
6. (Spreadsheet Problem) You are given that the random variable  $X$  is distributed as a Geometric distribution with parameters  $\beta = 3$ . Calculate:
  - a.  $\Pr[X \leq 5]$
  - b.  $\Pr[3 \leq X \leq 5]$
7. A Weibull Distribution with parameter  $\tau = 1$  becomes what distribution?
8. A random variable  $X$  has a density function  $f(x) = 4x(1+x^2)^{-3}$ , for  $x > 0$ . Determine the mode of  $X$ .

#### Chapter 3

9. Determine the following for a generalized uniform distribution covering the range from  $a$  to  $b$  where  $a \leq d \leq b$ :
  - a.  $E[X^k]$
  - b.  $E[X]$
  - c.  $\text{Var}(X)$
  - d.  $e(d)$
  - e.  $\text{VaR}_p$
  - f.  $\text{TVaR}_p$

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10. For the Pareto distribution, determine  $E[X]$ ,  $\text{Var}(X)$ , and the coefficient of variation in terms of  $\alpha$  and  $\theta$ .
11. For the Gamma distribution, determine  $E[X]$ ,  $\text{Var}(X)$ , the coefficient of variation, and skewness in terms of  $\alpha$  and  $\theta$ .
12. For the Exponential distribution, determine  $E[X]$ ,  $\text{Var}(X)$ , and  $e(d)$  in terms of  $\theta$ .
13. You are given:

$$F(x) = \begin{cases} x^3/27, & \text{for } 0 < x \leq 3 \\ 1, & \text{for } x > 3 \end{cases}$$

Calculate:

- a.  $E[X]$
  - b.  $\text{Var}(X)$
  - c.  $e(1)$
  - d.  $E[(X-1)_+]$
  - e.  $E[X \wedge 2]$
  - f. The Median
  - g. The standard deviation principle with  $k = 1$
  - h.  $\text{VaR}_{.80}$
  - i.  $\text{TVaR}_{.80}$
14. (Spreadsheet) If you roll two fair die,  $X$  is the sum of the dice. Calculate:
    - a.  $E[X]$
    - b.  $\text{Var}(X)$
    - c.  $e(4)$
    - d.  $E[(X-4)_+]$
    - e.  $E[X \wedge 10]$
    - f. The Mode
    - g.  $\pi_{20}$
  15. You are given a sample of 2, 2, 3, 5, 8. For this empirical distribution, determine:
    - a. The mean
    - b. The variance
    - c. The standard deviation
    - d. The coefficient of variation
    - e. The skewness
    - f. The kurtosis
    - g.  $\text{VaR}_{.80}$
    - h.  $\text{TVaR}_{.80}$

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16. \* Losses follow a Pareto distribution has parameters of  $\alpha = 7$  and  $\theta = 10,000$ . Calculate  $e(5,000)$
17. The amount of an individual claim has a Pareto distribution with  $\theta = 8000$  and  $\alpha = 9$ . Use the central limit theorem to approximate the probability that the sum of 500 independent claims will exceed 550,000.
18. Lifetimes of an iPod follows a Single Parameter Pareto distribution with  $\alpha > 1$  and  $\theta = 4$ . The expected lifetime of an iPod is 8 years.

Calculate the probability that the lifetime of an iPod is at least 6 years.

19. You are given that  $F_X(x) = 1 - (100/x)^4$  for  $x \geq 100$ .

You are also given that  $f_Y(y)$  is:

$y$	$f_Y(y)$
100	0.4
200	0.3
300	0.2
400	0.1

Calculate  $Var(Y) - Var(X)$ .

20. A company has 50 employees whose dental expenses are mutual independent. For each employee, the company reimburses 100% of the dental expenses. The dental expense for each employee is distributed as follows:

Expense	Probability
0	0.5
100	0.3
400	0.1
900	0.1

Using the normal approximation, calculate the 95<sup>th</sup> percentile of the cost to the company.

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### Chapter 4

21. The distribution function for losses from your renter's insurance is the following:

$$F(x) = 1 - 0.8[1000/(1000+x)]^5 - 0.2[12000/(12000+x)]^3$$

Calculate:

- a.  $E[X]$
  - b.  $\text{Var}(X)$
  - c. Use the normal approximation to determine the probability that the sum of 100 independent claims will not exceed 200,000.
22. \* X has a Burr distribution with parameters  $\alpha = 1$ ,  $\gamma = 2$ , and  $\theta = 1000^{0.5}$ . Y has a Pareto distribution with parameters  $\alpha = 1$  and  $\theta = 1000$ . Z is a mixture of X and Y with equal weights on each component. Determine the median of Z.
23. The random variable X is distributed as a Pareto distribution with parameters  $\alpha$  and  $\theta$ .  $E(X) = 1$  and  $\text{Var}(X) = 3$ . The random variable Y is equal to  $2X$ . Calculate the  $\text{Var}(Y)$ .
24. \* Claim severities are modeled using a continuous distribution and inflation impacts claims uniformly at an annual rate of  $s$ . Which of the following are true statements regarding the distribution of claim severities after the effect of inflation?
- i. An exponential distribution will have a scale parameter of  $(1+s)\theta$ .
  - ii. A Pareto distribution will have scale parameters  $(1+s)\alpha$  and  $(1+s)\theta$ .
  - iii. An Inverse Gaussian distribution will have a scale parameter of  $(1+s)\theta$ .
25. \* The aggregate losses of Eiffel Auto Insurance are denoted in euro currency and follow a Lognormal distribution with  $\mu = 8$  and  $\sigma = 2$ . Given that 1 euro = 1.3 dollars, determine the lognormal parameters for the distribution of Eiffel's losses in dollars.

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### Chapter 5

26. (Spreadsheet) Calculate  $\Gamma(1.2)$ .
27. The random variable  $X$  is the number of dental claims in a year and is distributed as a gamma distribution given parameter  $\theta$  and with parameter  $\alpha = 1$ .  $\theta$  is distributed uniformly between 1 and 3. Calculate  $E(X)$  and  $\text{Var}(X)$ .
28. A dental insurance company has 1000 insureds. Assume the number of claims from each insured is independent. Using the information in Problem 27 and the normal approximation, calculate the probability that the Company will incur more than 2100 claims.
29. \* Let  $N$  have a Poisson distribution with mean  $\Lambda$ . Let  $\Lambda$  have a uniform distribution on the interval  $(0,5)$ . Determine the unconditional probability that  $N \geq 2$ .

### Chapter 6

30. The number of hospitalization claims in a year follows a Poisson distribution with a mean of  $\lambda$ . The probability of exactly three claims during a year is 60% of the probability that there will be 2 claims. Determine the probability that there will be 5 claims.
31. If the number of claims is distributed as a Poisson distribution with  $\lambda = 3$ , calculate:
  - a.  $\Pr(N = 0)$
  - b.  $\Pr(N = 1)$
  - c.  $\Pr(N = 2)$
  - d.  $E(N)$
  - e.  $\text{Var}(N)$
32. If the number of claims is distributed as a zero truncated Poisson distribution with  $\lambda = 3$ , calculate:
  - a.  $\Pr(N = 0)$
  - b.  $\Pr(N = 1)$
  - c.  $\Pr(N = 2)$
  - d.  $E(N)$
  - e.  $\text{Var}(N)$

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33. If the number of claims is distributed as a zero modified Poisson distribution with  $\lambda = 3$  and  $p_0^M = 0.5$ , calculate:
- $\Pr(N = 0)$
  - $\Pr(N = 1)$
  - $\Pr(N = 2)$
  - $E(N)$
  - $\text{Var}(N)$
34. If the number of claims is distributed as a Geometric distribution with  $\beta = 3$ , calculate:
- $\Pr(N = 0)$
  - $\Pr(N = 1)$
  - $\Pr(N = 2)$
  - $E(N)$
  - $\text{Var}(N)$
35. (Spreadsheet) If the number of claims is distributed as a Binomial distribution with  $m = 6$  and  $q = 0.4$ , calculate:
- $\Pr(N = 0)$
  - $\Pr(N = 1)$
  - $\Pr(N = 2)$
  - $E(N)$
  - $\text{Var}(N)$
36. (Spreadsheet) If the number of claims is distributed as a Negative Binomial distribution with  $\gamma = 3$  and  $\beta = 2$ , calculate:
- $\Pr(N = 0)$
  - $\Pr(N = 1)$
  - $\Pr(N = 2)$
  - $E(N)$
  - $\text{Var}(N)$
37. \* The Independent Insurance Company insures 25 risks, each with a 4% probability of loss. The probabilities of loss are independent. What is the probability of 4 or more losses in the same year? (Hint: Use the binomial distribution.)
38. \* You are given a negative binomial distribution with  $\gamma = 2.5$  and  $\beta = 5$ . For what value of  $k$  does  $p_k$  take on its largest value?
39. \*  $N$  is a discrete random variable from the  $(a, b, 0)$  class of distributions. The following information is known about the distribution:
- $P(N = 0) = 0.327680$
  - $P(N = 1) = 0.327680$
  - $P(N = 2) = 0.196608$
  - $E[N] = 1.25$

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Based on this information, which of the following are true statements?

- I.  $P(N = 3) = 0.107965$
- II.  $N$  is from a binomial distribution.
- III.  $N$  is from a Negative Binomial Distribution.

40. \* You are given:

- Claims are reported at a Poisson rate of 5 per year.
- The probability that a claim will settle for less than 100,000 is 0.9.

What is the probability that no claim of 100,000 or more will be reported in the next three years?

41.  $N$  is distributed as a zero modified geometric distribution with  $\beta = 3$  and  $p_0^M = 0.5$ . Calculate:

- a.  $e(2)$
- b.  $Var[N \wedge 3]$
- c.  $E[(N-2)_+]$

42.  $N$  is the random variable representing the number of claims under homeowners insurance.  $N$  is distributed as a zero modified geometric distribution with  $\beta = 2$ .  $p_4^M = 2/27$ . Calculate  $p_2^M$ .

43. Under an unmodified geometric distribution,  $Var[N] = 20$ .

Under a zero-modified geometric distribution,  $Var[N] = 20.25$ .

The parameter  $\beta$  is the same for both distributions.

Calculate  $p_0^M$ .

### Chapter 8

44. Losses are distributed exponentially with  $\theta = 1000$ . Losses are subject to an ordinary deductible of 500. Calculate:

- a.  $f_{Y^L}(y)$
- b.  $F_{Y^L}(y)$
- c.  $E[Y^L]$
- d. The loss elimination ratio

45. Losses are distributed exponentially with parameter  $\theta = 1000$ . Losses are subject to an ordinary deductible of 500. Calculate:

- a.  $f_{Y^P}(y)$
- b.  $F_{Y^P}(y)$

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- c.  $E[Y^P]$
46. Losses are distributed exponentially with  $\theta = 1000$ . Losses are subject to a franchise deductible of 500. Calculate:
- $f_{Y^L}(y)$
  - $F_{Y^L}(y)$
  - $E[Y^L]$
  - The loss elimination ratio
47. Losses are distributed exponentially with parameter  $\theta = 1000$ . Losses are subject to a franchise deductible of 500. Calculate:
- $f_{Y^P}(y)$
  - $F_{Y^P}(y)$
  - $E[Y^P]$
48. Last year, losses were distributed exponentially with  $\theta = 1000$ . This year losses are subject to 10% inflation. Losses in both years are subject to an ordinary deductible of 500. Calculate the following for this year:
- $E(Y^L)$
  - $E(Y^P)$

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49. \* Losses follow an exponential distribution with parameter  $\theta$ . For a deductible of 100, the expected payment per loss is 2000. What is the expected payment per loss in terms of  $\theta$  for a deductible of 500? (Note: Assume it is an ordinary deductible.)

50. \* In year 2007, claim amounts have the following Pareto distribution

$$F(x) = 1 - (800/[x+800])^3$$

The annual inflation rate is 8%. A franchise deductible of 300 will be implemented in 2008. Calculate the loss elimination ratio of the franchise deductible.

51. (Spreadsheet) If you roll two fair die,  $X$  is the sum of the dice. Let  $X$  represent the distribution of losses from a particular event. If an ordinary deductible of 3 is applied, calculate:

- a.  $E(Y^L)$
- b.  $E(Y^P)$
- c.  $\text{Var}(Y^L)$
- d.  $\text{Var}(Y^P)$

52. Losses are distributed uniformly between 0 and 100,000. An insurance policy which covers the losses has a 10,000 deductible and an 80,000 upper limit. The upper limit is applied prior to applying the deductible. Calculate:

- a.  $E(Y^L)$
- b.  $E(Y^P)$

53. Last year, losses were distributed exponentially with  $\theta = 1000$ . This year losses are subject to 25% inflation. Losses in both years are subject to an ordinary deductible of 500, an upper limit of 4000, and coinsurance where the company pays 80% of the claim. Calculate the following for this year:

- a.  $E(Y^L)$
- b.  $E(Y^P)$

54. \* An insurance company offers two types of policies: Type Q and Type R. Type Q has no deductible, but has a policy limit of 3000. Type R has no limit, but has an ordinary deductible of  $d$ . Losses follow a Pareto distribution with  $\theta = 2000$  and  $\alpha = 3$ .

Calculate the deductible  $d$  such that both policies have the same expected cost per loss.

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55. \* Well Traveled Insurance Company sells a travel insurance policy that reimburses travelers for any expense incurred for a planned vacation that is cancelled because of airline bankruptcies. Individual claims follow a Pareto distribution with  $\alpha = 2$  and  $\theta = 500$ . Well Traveled imposes a limit of 1000 on each claim. If a planned policyholder's planned vacation is cancelled due to airline bankruptcy and he or she has incurred more than 1000 of expenses, what is the expected non-reimbursable amount of the claim?
56. If  $X$  is uniformly distributed on from 0 to  $b$ . An ordinary deductible of  $d$  is applied. Calculate:
- $E[Y^L]$
  - $\text{Var}[Y^L]$
  - $E[Y^P]$
  - $\text{Var}[Y^P]$
57. Losses follow a Pareto distribution with  $\alpha = 3$  and  $\theta = 200$ . A policy covers the loss except for a franchise deductible of 50.  $X$  is the random variable representing the amount paid by the insurance company per payment. Calculate the expected value of the  $X$ .
58. Losses in 2007 are uniformly distributed between 0 and 100,000. An insurance policy pays for all claims in excess of an ordinary deductible of \$20,000. For 2008, claims will be subject to uniform inflation of 20%. The Company implements an upper limit  $u$  (without changing the deductible) so that the expected cost per loss in 2008 is the same as the expected cost per loss in 2007. Calculate  $u$ .
59. Losses follow an exponential distribution with a standard deviation of 100. An insurance company applies an ordinary policy deductible  $d$  which results in a Loss Elimination Ratio of 0.1813. Calculate  $d$ .
60. Losses are distributed following the **single parameter** Pareto with  $\alpha = 4$  and  $\theta = 90$ . An insurance policy is issued with an ordinary deductible of 100. Calculate the  $\text{Var}(Y^L)$ .

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61. Losses follow a Pareto distribution with  $\alpha = 5$  and  $\theta = 2000$ . A policy pays 100% of losses from 500 to 1000. In other words, if a loss occurs for less than 500, no payment is made. If a loss occurs which exceeds 1000, no payment is made. However, if a loss occurs which is between 500 and 1000, then the entire amount of the loss is paid. Calculate  $E(Y^P)$ .
62. \* You are given the following:
- Losses follow a lognormal distribution with parameters  $\mu = 10$  and  $\sigma = 1$ .
  - For each loss less than or equal to 50,000, the insurer makes no payment.
  - For each loss greater than 50,000, the insurer pays the entire amount of the loss up to the maximum covered loss of 100,000.
- Determine the expected amount of the loss.
63. You are given the following:
- Losses follow a distribution prior to the application of any deductible with a mean of 2000.
  - The loss elimination ratio at a deductible of 1000 is 0.3.
  - 60% of the losses in number are less than the deductible of 1000.
- Determine the average size of the loss that is less than the deductible of 1000.
64. Losses follow a Pareto distribution with  $\alpha = 5$  and  $\theta = 2000$ . An insurance policy covering these losses has a deductible of 100 and makes payments directly to the physician. Additionally, the physician is entitled to a bonus if the **loss** is less than 500. The bonus is 10% of the difference between 500 and the amount of the loss.

The following table should help clarify the arrangement:

Amount of Loss	Loss Payment	Bonus
50	0	45
100	0	40
250	150	25
400	300	10
500	400	0
1000	900	0

Calculate the expected total payment (loss payment plus bonus) from the insurance policy to the physician per loss.

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### Chapter 9

65. \* For an insured,  $Y$  is the random variable representing the total amount of time spent in a hospital each year. The distribution of the number of hospital admissions in a year is:

Number of Admissions	Probability
0	0.60
1	0.30
2	0.10

The distribution of the length of each stay for an admission follows a Gamma distribution with  $\alpha = 1$  and  $\theta = 5$ .

Calculate  $E[Y]$  and  $\text{Var}[Y]$ .

66. An automobile insurer has 1000 cars covered during 2007. The number of automobile claims for each car follows a negative binomial distribution with  $\beta = 1$  and  $\gamma = 1.5$ . Each claim is distributed exponentially with a mean of 5000. Assume that the number of claims and the amount of the loss are independent and identically distributed.

Using the normal distribution as an approximating distribution of aggregate losses, calculate the probability that losses will exceed 8 million.

67. For an insurance company, each loss has a mean of 100 and a variance of 100. The number of losses follows a Poisson distribution with a mean of 500. Each loss and the number of losses are mutually independent.

The loss ratio for the insurance company is defined as the ratio of aggregate losses to the total premium collected.

The premium collected is 110% of the expected aggregate losses.

Using the normal approximation, calculate the probability that the loss ratio will exceed 95%.

68. The number of claims follows a Poisson distribution with a mean of 3. The distribution of claims is  $f_X(1) = 1/3$  and  $f_X(2) = 2/3$ . Calculate  $f_S(4)$ .

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69. (Spreadsheet) The number of claims and the amount of each claim is mutually independent. The frequency distribution is:

$n$	$p_n$
1	0.05
2	0.20
3	0.30
4	0.25
5	0.15
6	0.05

The severity distribution is:

$x$	$f_X(x)$
100	0.25
200	0.20
300	0.15
400	0.12
500	0.10
600	0.08
700	0.06
800	0.04

Calculate  $f_S(x)$ . (See Example 9.5 in the book for an example.)

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70. You are given the following table for aggregate claims:

<b>s</b>	<b>F<sub>s</sub>(s)</b>	<b>E[(S-d)<sub>+</sub>]</b>
0		
100		
200	0.50	
300	0.65	60
400	0.75	
500		
600		

- Losses can only occur in multiples of 100. Calculate the net stop loss premium for stop loss insurance covering losses in excess of 325.
71. \* Losses follow a Poisson frequency distribution with a mean of 2 per year. The amount of a loss is 1, 2, or 3 with each having a probability of 1/3. Loss amounts are independent of the number of losses and from each other.
- An insurance policy covers all losses in a year subject to an annual aggregate deductible of 2. Calculate the expected claim payments for this insurance policy.
72. \* An insurance portfolio produces N claims with the following distribution:

<b>n</b>	<b>P(N=n)</b>
0	0.1
1	0.5
2	0.4

Individual claim amounts have the following distribution:

<b>x</b>	<b>f<sub>x</sub>(x)</b>
0	0.7
10	0.2
20	0.1

Individual claim amounts and counts are independent.

Stop Loss insurance is purchased with an aggregate deductible of 400% of expected claims.

Calculate the net stop loss premium.

73. Losses are modeled assuming that the amount of all losses is 40 and that the number of losses follows a geometric distribution with a mean of 4. Calculate the net stop loss premium for coverage with an aggregate deductible of 100.

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74. \* An aggregate claim distribution has the following characteristics:  
 $P[S = i] = 1/6$  for  $i = 1, 2, 3, 4, 5,$  or  $6$ . A stop loss insurance with a deductible amount of  $d$  has an expected insurance payment of  $1.5$

Determine  $d$ .

75. For an automobile policy, the severity distribution is Gamma with  $\alpha = 3$  and  $\theta = 1000$ . The number of claims in a year is distributed as follows:

Number of Claims	Probability
1	0.65
2	0.20
3	0.10
4	0.05

Calculate:

- a.  $E[S]$
  - b.  $Var[S]$
  - c.  $F_S(10,000)$
76. (Spreadsheet) You are given that the number of losses follow a Poisson with  $\lambda = 6$ . Losses are distributed as follows:  $f_X(1) = f_X(2) = f_X(4) = 1/3$ . Find  $f_S(s)$  for total losses of 100 or less.
77. On a given day, a doctor provides medical care for  $N_A$  adults and  $N_C$  children. Assume that  $N_A$  and  $N_C$  have Poisson distributions with parameters 3 and 2 respectively. The distributions for the length of care per patient are as follows:

Time	Adult	Child
1 hour	0.4	0.9
2 hours	0.6	0.1

Let  $N_A$ ,  $N_C$ , and the lengths of stay for all patients be independent. The doctor charges 200 per hour. Determine the probability that the office income on a given day will be less than or equal to 800.

78. (Spreadsheet) Losses follow a Pareto distribution with  $\alpha = 3$  and  $\theta = 1000$ .
- a. Using the Method of Rounding with a span of 100, calculate  $f_j$  for  $0 \leq j \leq 1000$ . Calculate the mean of the discretized distribution. Compare this mean to the actual mean.
  - b. Using the Method of Local Moment Matching and matching the mean (with  $k = 1$ ) and a span of 100, calculate  $f_j$  for  $0 \leq j \leq 1000$ . Calculate the mean of the discretized distribution. Why does this mean not equal the actual mean?

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79. Using the example that was handed out in class, find  $f_S(6)$  and  $f_S(7)$ .
80. \* You are given:
- S has a compound Poisson distribution with  $\lambda = 2$ ; and
  - Individual claim amounts  $x$  are distributed as follows:  
 $f_X(1) = 0.4$  and  $f_X(2) = 0.6$

Determine  $f_S(4)$ .

81. \* Aggregate claims  $S$  has a compound Poisson distribution with discrete individual claim amount distributions of  $f_X(1) = 1/3$  and  $f_X(3) = 2/3$ .

Also,  $f_S(4) = f_S(3) + 6f_S(1)$ .

Calculate the  $\text{Var}(S)$ .

82. \* For aggregate claims  $S$ , you are given:

$$a. f_S(X) = \sum_{n=0}^{\infty} f_X^{*n}(x) \cdot \frac{e^{-50} (50)^n}{n!}$$

- b. Losses are distributed as follows:

$x$	$f_X(x)$
1	0.4
2	0.5
3	0.1

Determine  $\text{Var}(S)$

83. With no deductible, the number of payments for losses under warranty coverage for an iPhone follows a negative binomial distribution with a mean of 0.25 and a variance of 0.375.

The company imposes a deductible of  $d$  such that the number of expected payments for losses is reduced to 50% of the prior expected payments.

Calculate the  $\text{Var}(N^P)$  after the imposition of the deductible.

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84. \* Prior to the application of any deductible, aggregate claim counts during 2005 followed a Poisson with  $\lambda = 14$ . Similarly, individual claim sizes followed a Pareto with  $\alpha = 3$  and  $\theta = 1000$ .

Annual inflation for the claim sizes is 10%.

All policies in 2005 and 2006 are subject to a 250 ordinary deductible.

Calculate the increase in the number of claims that exceed the deductible in 2006 when compared to 2005.

85. Purdue University has decided to provide a new benefit to each class at the university. Each class will be provided with group life insurance. Students in each class will have 10,000 of coverage while the professor for the class will have 20,000. STAT 479 has 27 students all age 22 split 12 males and 15 females. The class also has an aging professor. The probability of death for each is listed below:

Age	Gender	Probability of Death
22	Male	0.005
22	Female	0.003
58	Male	0.020

The insurance company providing the coverage charges a premium equal to the expected claims plus one standard deviation. Calculate the premium.

86. Purdue University has decided to provide a new benefit to each class at the university. Each class will be provided with group life insurance. Students in each class will have 10,000 of coverage while the professor for the class will have 20,000. STAT 479 has 27 students all age 22 split 12 males and 15 females. The class also has an aging professor. The probability of death for each is listed below:

Age	Gender	Probability of Death
22	Male	0.005
22	Female	0.003
58	Male	0.020

Purdue purchases a Stop Loss Policy such with an aggregate deductible of 20,000. Calculate the net premium that Purdue will pay for the stop loss coverage.

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87. \* An insurer provides life insurance for the following group of independent lives:

<b>Number of Lives</b>	<b>Death Benefit</b>	<b>Probability of Death</b>
100	1	0.01
200	2	0.02
300	3	0.03

The insurer purchases reinsurance with a retention of 2 on each life.

The reinsurer charges a premium of  $H$  equal to the its expected claims plus the standard deviation of its claims.

The insurer charges a premium of  $G$  which is equal to its expected retained claims plus the standard deviation of the retained claims plus  $H$

Calculate  $G$ .

88.  $X_1$ ,  $X_2$ , and  $X_3$  are mutually independent random variables. These random variables have the following probability functions:

<b><math>x</math></b>	<b><math>f_1(x)</math></b>	<b><math>f_2(x)</math></b>	<b><math>f_3(x)</math></b>
0	0.6	0.4	0.1
1	0.4	0.3	0.4
2	0.0	0.2	0.5
3	0.0	0.1	0.0

Calculate  $f_S(x)$ .

## Homework Problems Stat 479

89. \* Two portfolios of independent insurance policies have the following characteristics:

<b>Portfolio A</b>			
<b>Class</b>	<b>Number in Class</b>	<b>Probability of Claim</b>	<b>Claim Amount Per Policy</b>
1	2000	0.05	1
2	500	0.10	2

<b>Portfolio B</b>				
<b>Class</b>	<b>Number in Class</b>	<b>Probability of Claim</b>	<b>Claim Amount Distribution</b>	
			<b>Mean</b>	<b>Variance</b>
1	2000	0.05	1	1
2	500	0.10	2	4

The aggregate claims in the portfolios are denoted by  $S_A$  and  $S_B$ , respectively.

Calculate  $\text{Var}[S_B]/\text{Var}[S_A]$ .

90. \* An insurance company is selling policies to individuals with independent future lifetimes and identical mortality profiles. For each individual, the probability of death by all causes is 0.10 and the probability of death due to an accident is 0.01. Each insurance policy pays a benefit of 10 for an accidental death and 1 for non-accidental death.

The company wishes to have at least 95% confidence that premiums with a relative security loading of 0.20 are adequate to cover claims. (In other words, the premium is  $1.20E(S)$ .)

Using the normal approximation, determine the minimum number of policies that must be sold.

**Homework Problems**  
**Stat 479**  
**Answers**

1. Not Provided
2.  $\frac{3}{4}$
3. 2
4. 100/9999
5.
  - a. 0.725
  - b. 0.093
6.
  - a. .822
  - b. 0.244
7. No Answer Given
8.  $1/\sqrt{5}$
9.
  - a.  $(b^{k+1} - a^{k+1})/(b-a)(k+1)$
  - b.  $(b+a)/2$
  - c.  $(b-a)^2/12$
  - d.  $(b-d)/2$
  - e.  $(1-p)a+pb$
  - f.  $[b+(1-p)a+pb]/2$
10.
  - a.  $\theta/(\alpha-1)$
  - b.  $\alpha\theta^2/[(\alpha-1)^2(\alpha-2)]$
  - c.  $[\alpha/(\alpha-2)]^{0.5}$
11.
  - a.  $\theta\alpha$
  - b.  $\theta^2\alpha$
  - c.  $1/\sqrt{\alpha}$
  - d.  $2/\sqrt{\alpha}$
12.
  - a.  $\theta$
  - b.  $\theta^2$
  - c.  $\theta$
13.
  - a. 2.25
  - b. 27/80
  - c. 17/13
  - d. 34/27
  - e. 50/27
  - f. 2.3811
  - g. 2.83
  - h. 2.785
  - i. 2.895

**Homework Problems**  
**Stat 479**  
**Answers**

- 14.
- a. 7
  - b. 5.8333
  - c. 3.7333
  - d. 3.1111
  - e. 6.8888
  - f. 7
  - g. 5
- 15.
- a. 4
  - b. 5.2
  - c. 2.28
  - d. 0.57
  - e. 0.8096
  - f. 2.145
  - g. 5
  - h. 8
16. 2500
17. 0.0244
18. 4/9
19. 7777.78
20. 11,173
- 21.
- a. 1400
  - b. 26,973,333
  - c. 87.7%
22. 100
23. 12
24. i only. Be sure to indicate how to make the others true.
25.  $\mu = 8.26$  and  $\sigma = 2.00$
26. 0.918169
27. 2 and 14/3
28. 7.2%
29. 0.6094
30. 2.6%
- 31.
- a. 0.0498
  - b. 0.1494
  - c. 0.2240
  - d. 3
  - e. 3
- 32.
- a. 0
  - b. 0.1572

## Homework Problems Stat 479

- c. 0.2358
- d. 3.1572
- e. 2.6609

### Answers

- 33.
- a. 0.5000
  - b. 0.0786
  - c. 0.1179
  - d. 1.5786
  - e. 3.8224
- 34.
- a. 0.2500
  - b. 0.1875
  - c. 0.1406
  - d. 3
  - e. 12
- 35.
- a. 0.0467
  - b. 0.1866
  - c. 0.3110
  - d. 2.4
  - e. 1.44
- 36.
- a. 0.0370
  - b. 0.0741
  - c. 0.0988
  - d. 6
  - e. 18
37. 0.0165
38. 7
39. Statement III is only true statement
40. 0.2231
- 41.
- a. 4
  - b. 1.6943
  - c. 1.125
42. 1/6
43. 0.1
- 44.
- a.  $f_{YL}(y) = 1 - e^{-0.5}, y = 0$   
 $f_{YL}(y) = 0.001e^{-(0.001y + 0.5)}, y > 0$
  - b.  $F_{YL}(y) = 1 - e^{-0.5}, y = 0$   
 $F_{YL}(y) = 1 - e^{-(.001 y + 0.5)}, y > 0$

**Homework Problems**  
**Stat 479**

c.  $1000e^{-0.5}$

d.  $1 - e^{-0.5}$

45.

a.  $f_{YP}(y) = 0.001e^{-0.001y}$

b.  $F_{YP}(y) = 1 - e^{-0.001y}$

c. 1000

46.

a.  $f_{YL}(y) = 1 - e^{-0.5}, y = 0$   
 $f_{YL}(y) = 0.001e^{-0.001y}, y > 500$

b.  $F_{YL}(y) = 1 - e^{-0.5}, y = 0$   
 $F_{YL}(y) = 1 - e^{-0.001y}, y > 500$

c.  $1500e^{-0.5}$

d.  $1 - 1.5e^{-0.5}$

47.

a.  $f_{YP}(y) = 0.001e^{-0.001y+0.5}$

b.  $F_{YP}(y) = 1 - e^{-0.001y+0.5}$

c. 1500

**Homework Problems**  
**Stat 479**  
**Answers**

48.  
a.  $1100e^{-(0.5/1.1)}$   
b. 1100
49.  $2000e^{-400/\theta}$
50. 0.165
51.  
a. 4.0278  
b. 4.39  
c. 5.58  
d. 4.48
52.  
a. 38,500  
b. 42,777.78
53.  
a. 629.56  
b. 939.19
54. 182.18
55. 1500
56.  
a.  $(b-d)^2/2b$   
b.  $[(b-d)^3/3b] - [(b-d)^2/2b]^2$   
c.  $(b-d)/2$   
d.  $(b-d)^2/12$
57. 175
58. 71,833.62
59. 20
60. 1708.70
61. 705.06
62. 16,224
63. 333.33
64. 431.83
65. 2.5 and 23.75
66. 0.068
67. 0.1587

## Homework Problems Stat 479

- 68. 0.151436
- 69. None provided
- 70. 51.25
- 71. 2.360894
- 72. 0.224
- 73. 92.16
- 74. 2.25
- 75.
  - a. 4,650
  - b. 11,377,500
  - c.
- 76. No Answer Provided
- 77. 0.2384
- 78.
  - a. Mean = 498.61
  - b. Mean = 499.85
- 79. 0.106978 and 0.060129
- 80. 0.1517
- 81. 76
- 82. 165
- 83. 0.15625
- 84. 0.406
- 85. 5,727
- 86. 22.60
- 87. 46.13
- 88.

x	$f_s(x)$
0	0.024
1	0.130
2	0.280
3	0.280
4	0.180
5	0.086
6	0.020

- 89. 2.1
- 90. 1975
- 91.