STAT 479
Test 2
Spring 2017

March 30, 2017

1. (8 points) Combs Life Insurance Company completes a three year mortality study using the following data:

Life	Date of Entry	Date of Exit	Reason for Exit
1	0	0.1	Lapse
2	0	0.2	Death
3	0	0.2	Lapse
4	0	0.3	Death
5	0	0.3	Lapse
6	0	0.4	Lapse
7	0	0.5	Lapse
8	0	0.5	Death
9	0	0.5	Death
10	0	0.5	Lapse
11	0	1.0	Death
12	0	3.0	Expiry of Policy
13	0	3.0	Expiry of Policy
14	0	3.0	Expiry of Policy
15	0	3.0	Expiry of Policy
16	0.2	0.3	Lapse
17	0.3	0.5	Death
18	0.3	3.0	Expiry of Policy
19	0.5	3.0	Expiry of Policy
20	2.0	2.5	Death

Calculate a 90% linear confidence interval for $H_{20}(0.5)$ using the Nelson-Åalen estimator.

Not an Applicable Question.

2. (7 points) Wang Dental Company has collected the following data regarding the last 100 dental claims:

Amount of Claim	**Number of Claims**
0 – 100	27
100 – 250	58
250 – 500	10
500 – 1000	3
1000 – 10,000	2

Using the Ogive, find the median of this distribution.

Not an Applicable Question.

3. Chen Casualty Company provides warranty insurance on iPhones.

   The number of claims under a policy in a one year period for this coverage is distributed as a Geometric distribution with $\beta = 0.4$.

   The amount of each claim is distributed as an Exponential distribution with $\theta = 250$.

   Let S be the random variable that is the aggregate claims under a policy for a one year period.

   a. (3 points) Calculate $E[S]$.

      Solution:

      $$E[S] = E[N]E[X] = \beta \cdot \theta = (0.4)(250) = 100$$

   b. (4 points) Calculate $Var[S]$.

      Solution:

      $$Var[S] = E[N]Var[X] + Var[N]\left(E[X]\right)^2 = (0.4)(250)^2 + (0.56)(250)^2 = 60,000$$

      $$Var[X] = \theta^2 = (250)^2 \qquad Var[N] = \beta(1+\beta) = 0.4(1.4) = 0.56$$

   c. (3 points) Calculate $f_S(200)$.

      Solution:

      If N is distributed as a geometric with parameter β and X is distributed as an exponential with parameter θ, then S is distributed as a two point mixture:

      $f_S(0)$ has a weight of $\dfrac{1}{1+\beta}$. For any other value of S, it is distributed as an exponential with a mean of $\theta(1+\beta)$ with a weight of $\dfrac{\beta}{1+\beta}$.

      $$f_S(200) = \frac{\beta}{1+\beta}\left[f_X(200) \text{ for an exponential with a mean of } \theta(1+\beta) = (250)(1.4) = 350\right]$$

      $$f_S(200) = \frac{0.4}{1.4}\left[\frac{e^{-200/350}}{350}\right] = 0.000461$$

(Continued from Prior Page) Chen sells 10,000 policies to 10,000 independent iPhone owners.

d. (4 points) Calculate the probability that the total claims that Chen will have to pay will exceed 1,050,000.

Solution:

$$E[Port) = (10,000)(100) = 1,000,000$$

$$Var(Port) = (10,000)(60,000) = 600,000,000$$

$$\Pr(S > 1,050,000) = \Pr\left(Z > \frac{1,050,000 - 1,000,000}{\sqrt{600,000,000}} \right) = \Pr(Z > 2.04) = 1 - 0.9793 = 0.0207$$

4. (6 points) Burnell Insurance Company sells a product liability policy. The number of claims in a single year under this policy has the following distribution:

Number of Claims	Probability
0	0.1
1	0.2
2	0.3
3	0.4

The amount of each claim is distributed as follows:

Amount of Claim	Probability
100	0.15
250	0.25
400	0.38
500	0.22

Calculate $f_S(500)$.

Solution:

$$f_S(500) = \Pr(N=1)\Pr(X=500) + \Pr(N=2)\Pr(X=250,250) + \Pr(N=2)\Pr(X=100,400) + \Pr(N=2)\Pr(X=400,100)$$

$$= (0.2)(0.22) + (0.3)(0.25)^2 + (0.3)(2)(0.15)(0.38) = 0.09695$$

5. (12 points) The Cao Car Company sells automobile insurance. Claims for automobile insurance are distributed as a Pareto distribution with $\alpha = 3$ and $\theta = 10,000$.

Yuanzheng, the owner of Cao, wants to create a discrete distribution for the claims using the above information. Yuanzheng wants to use a span of 1000. However, he is not sure which method to use to calculate the discrete distribution.

He calls in his actuaries to discuss creating a discrete distribution. He instructs Shivam to use the Method of Rounding to find f_2 for the discrete distribution. f_2 is the probability assigned to 2000 in the discrete distribution.

Yuanzheng also asks the company's consulting actuary, Tracy, to calculate the value of f_2 using the Method of Moment Matching where the results will match the first moment.

Calculate the values of f_2 determined by Shivam and Tracy.

Solution:

Shivam

$$f_2 = F(2500) - F(1500) = \left[1 - \left(\frac{10,000}{2500 + 10,000}\right)^3\right] - \left[1 - \left(\frac{10,000}{1500 + 10,000}\right)^3\right] = 0.14552$$

Tracy

$$f_2 = \frac{2E[X \wedge 2000] - E[X \wedge 1000] - E[X \wedge 3000]}{h}$$

$$= \frac{2\left[\frac{10,000}{3-1}\left(1 - \left[\frac{10,000}{2000 + 10,000}\right]^{3-1}\right)\right] - \left[\frac{10,000}{3-1}\left(1 - \left[\frac{10,000}{1000 + 10,000}\right]^{3-1}\right)\right] - \left[\frac{10,000}{3-1}\left(1 - \left[\frac{10,000}{3000 + 10,000}\right]^{3-1}\right)\right]}{1000}$$

$$= \frac{(5000)\left[\left[\frac{10,000}{11,000}\right]^2 + \left[\frac{10,000}{13,000}\right]^2 - 2\left[\frac{10,000}{12,000}\right]^2\right]}{1000} = 0.14637$$

6. (6 points) The random variable X is distributed as an exponential distribution with a parameter of θ. We select a sample of three independent selections from this distribution - X_1 X_2 and X_3. We use the following estimator to estimate θ:

$$\theta = \frac{X_1}{3} + \frac{X_2}{3} + \frac{X_3}{3}$$

Calculate the Mean Square Error in this estimate. Your answer will be in terms of θ.

Not an Applicable Question.

7. (6 points) An urn contains four balls. Each ball has a unique number on it. The numbers on the balls are 1, 2, 3, and 4.

   Two balls are drawn from the urn and the average of the two numbers on the balls drawn are used to estimate the mean of the numbers on the balls in the urn prior to any being drawn.

   Calculate the Mean Square Error of this estimator.

   Not an Applicable Question.

8. You are given the following sample of claims:

   X: 5 8 8 10 12 15 15 20 24 30 N=10 $\sum X_i = 147$ $\sum X_i^2 = 2723$

   You want to use this data to complete hypothesis testing where your hypothesis is:

   H_0: The mean of the underlying distribution is 16.5.

   H_1 : The mean of the underlying distribution is less than 16.5

   a. (5 points) Calculate the z-factor for this hypothesis test.

   Not an Applicable Question.

   b. (2 points) State the critical value(s) at a 75% significance level and your conclusion regarding the hypothesis.

   Not an Applicable Question.

9. (9 points) Peilun has a farm where he takes care of abandoned dogs. Being an actuary, Peilun decided to do a mortality study on the dogs. He collects the following data on 100 dogs:

   a. There were 50 dogs in the farm at time 0.
   b. There were 10 dogs that entered the farm at time 1
   c. There were 20 dogs that entered the farm at time 3
   d. There were 5 dogs that entered the farm at time 4
   e. There were 15 dogs that entered the farm at time 5
   f. There were 13 dogs that were adopted and left the farm at time 2
   g. There were 10 dogs that were adopted and left the farm at time 3
   h. There were 7 dogs that were adopted and left the farm at time 4
   i. There were 25 dogs still alive at the end of 10 years.
   j. The remaining 45 dogs died as follows:

Number of Years till Death	**Number of Dogs Dying**
1	5
2	8
3	6
4	4
5	9
6	7
7	3
8	1
9	1
10	1

Let $\hat{S}(3)$ be the estimated probability of survivorship for 3 years for the dogs being studied based on the Kaplan Meier Product Limit Estimator.

Using the Greenwood approximation, calculate the $(10,000)Var[\hat{S}(3)]$.

Not an Applicable Question.