STAT 47900
Test 3
Spring 2017
May 3, 2017

1. Brandon has the following sample of five claims:

   20 35 45 55 70

   Brandon wants to develop a continuous distribution of the amount of claims using the Kernel Density Model using the uniform kernel with a span of 30.

   Calculate $\hat{F}(50)$.

   Not An Applicable Question.

2.

   a. Calculate the mean and variance for the distribution in Question 1.

      Not An Applicable Question.

b. Determine the mean and variance of the resulting distribution if Brandon had used the triangular kernel with a span of 30 with the data in Question 1.

   Not An Applicable Question.

3. A health insurance company has the following sample of five claims:

   500, 500, 700, 800, 1000

   The company wants to model their claims using a Gamma distribution.

   Calculate the parameters for the Gamma distribution using the Method of Moments.

   Not An Applicable Question.

4. The variance of a distribution will be estimated using:

$$\hat{\sigma}^2 = \frac{\sum (X - \bar{X})^2}{n}$$

The following sample is selected from the distribution:

5, 5, 8

Using the bootstrap method, estimate the mean square error in the above estimator.

Not An Applicable Question.

5. During the last calendar year, Purdue incurred 100 worker's compensation claims which are summarized in the following table:

Amount of Claim	Number of Claims
0 – 5000	13
5000-15,000	17
15,000-45,000	38
More than 45,000	32

Zihe wants to test the following hypothesis using the Chi Square Test at a 10% significance level:

H_0: Workers Compensation claims are uniformly distributed with θ = 60,000.
H_1: Workers Compensation claims are not uniformly distributed with θ = 60,000.

Calculate the Chi Square test statistic.

Solution:

Claim Amount	**Number of Claims** (O_j)	E_j	$\frac{(E_j - O_j)^2}{E_j}$
0 – 5000	13	$100\left(\frac{5000-0}{60,000}\right) = 8.333$	2.614
5000-15,000	17	$100\left(\frac{15,000-5000}{60,000}\right) = 16.667$	0.007
15,000-45,000	38	$100\left(\frac{45,000-15,000}{60,000}\right) = 50$	2.88
More than 45,000	32	$100\left(\frac{60,000-45,000}{60,000}\right) = 25$	1.96
		$\chi^2 \rightarrow$	7.461

Determine the critical value for this test.

Solution:

Degrees of freedom = 4 – 1 – 0 = 3 ➔ Critical Value = 6.251

State your conclusion with regard to the hypothesis.

Since χ^2 > 6.251, we reject H_0.

6. Using simulation, Tracy estimates the claim **payments** that will be made under a dental insurance policy. The policy has a \$50 deductible for each claim. The maximum that will be paid under the dental policy for all claims in a year is \$500.

   You are given that the number of claims is distributed as a Poisson distribution with $\lambda = 2$.

   The amount of each claim is distributed as an exponential distribution with $\theta = 200$.

   Using simulation, Tracy wants to estimate the total claims that will need to be paid under the dental policy. She does so by estimating the claims for each insured. Dali and Miao are the first two insureds. First, Tracy determines the number of claims for Dali and then the amount of each claim for Dali. Next, Tracy determines the number of claims for Miao. Finally, Tracy simulates the amount of each of Miao's claims.

   The random numbers used in the simulation are:

   0.20 0.55 0.85 0.50 0.30 0.95 0.45 0.60 0.95 0.75 0.05

   Calculate the amount paid for Dali and the amount paid for Miao.

   Solution:

$p_0 = e^{-2} = 0.1353$

$p_1 = \frac{e^{-2}(2)}{1!} = 0.2706$

$p_2 = \frac{e^{-2}(2)^2}{2!} = 0.2706$

$p_2 = \frac{e^{-2}(2)^3}{3!} = 0.1804$

==> Random Number 0 - 0.1353 ==> 0 claims
==> Random Number 0.1353 - 0.4060 ==> 1 claims
==> Random Number 0.4060 - 0.6767 ==> 2 claims
==> Random Number 0.6767 - 0.8797 ==> 3 claims
Amount of Claim
$u^* = 1 - e^{-x^*/200}$ ==> $e^{-x^*/200} = 1 - u^*$ ==> $-x^*/200 = \ln(1-u^*)$

$x^* = 200(-\ln(1-u^*))$

Dali

First Random number = 0.2 ==> 1 claim

Second random number = 0.55 ==> Amount of claim =$200(-\ln(1-0.55)) = 159.70$

Amount of payment is $159.70 - 50$ = 109.70.

Miao

Next Random number = 0.85 ==> 3 claims

Next random number = 0.50 ==> Amount of first claim =$200(-\ln(1-0.5)) = 138.63$
Next random number = 0.30 ==> Amount of second claim =$200(-\ln(1-0.3)) = 71.33$
Next random number = 0.95 ==> Amount of third claim =$200(-\ln(1-0.95)) = 599.15$

Amount of first payment is $138.63 - 50$ = 88.63
Amount of second payment is $71.33 - 50$ = 21.33
Amount of third payment is $599.15 - 50$ = 549.15. However, the total amount paid for Miao cannot exceed 500 so the amount accually paid will be $500 - 88.63 - 21.33 = 390.04$

7. Ye Fire Company has experienced the following four claims this year:

5000 25,000 50,000 60,000

The company's actuary, Tony, wants to test the following hypothesis using the Kolmogorov-Smirnov Test with a 5% significance level:

H_0: Claims are distributed as an exponential distribution with θ = 40,000.
H_1: Claims are not distributed as an exponential distribution with θ = 40,000.

Calculate the Kolmogorov-Smirnov test statistic.

Solution:

x	$F_n(x^-)$	$F_n(x)$	$F^*(x) = 1 - e^{-x/40,000}$	Absolute Value of Maximum Difference
5000	0	0.25	$1-e^{-5,000/40,000}$ =0.1175	0.1325
25,000	0.25	0.50	$1-e^{-25,000/40,000} = 0.4647$	0.2147
50,000	0.50	0.75	$1-e^{-50,000/40,000} = 0.7135$	0.2135
60,000	0.75	1.00	$1-e^{-60,000/40,000} = 0.7769$	0.2231

D =Maximum absolute value = 0.231

Calculate the critical value.

Solution:

Critical Value = $\dfrac{1.36}{\sqrt{4}} = 0.68$

State your conclusion with regard to Tony's hypothesis.

Solution:

Since D<0.68, we do not reject H_0

8. The following ten dental claim payments were made by Dowell Dental:

30, 50, 60, 100, 100, 120, 140, 150, 150, 150

Claims under Dowell Dental are subject to an upper limit of 150.

Dowell models claims using an Exponential distribution.

Determine the Maximum Likelihood Estimator for θ.

Solution:

$$MLE = \frac{Sum\ of\ \ Payments}{Uncensored\ Payments}$$

$$= \frac{(30+50+60+100+100+120+140+150+150+150}{7} = 150$$

9. Combs Dental Insurance Company has the following data from 1000 insureds:

Number of Claims in One Year	Number of Insureds
1	100
2	400
3	250
4	150
5	100

Combs believes that the number of claims in one year is distributed as a binomial distribution.

Use the method of moments to estimate the parameters for a binomial distribution.

Not An Applicable Question.

10.

a. State the principle of parsimony.

   Not An Applicable Question.

b. Which score based approach to determining a model is consistent with the principle of parsimony?

   Not An Applicable Question.

11. Chen Automobile Insurance Company has the following information regarding three auto claims:

    a. One claim is equal to 5000.
    b. Two claims exceed 5000.

    The claims are assumed to follow a Pareto distribution which α = 3.

    Calculate the Maximum Likelihood Estimator for θ.

    Solution:

$$L(\theta) = f(5000)\left[S(5000)\right]^2 = \left(\frac{3\theta^3}{(5000+\theta)^4}\right)\left(\frac{\theta}{5000+\theta}\right)^{(3)(2)} = \frac{3\theta^9}{(5000+\theta)^{10}}$$

$$l(\theta) = \ln(3) + 9\ln(\theta) - 10\ln(5000+\theta)$$

$$l'(\theta) = 0 + 9(\theta)^{-1} - 10(5000+\theta)^{-1} = 0 \Longrightarrow 9(5000+\theta) = 10(\theta) \Longrightarrow \theta = 45,000$$

12. You are given the following sample of five claim amounts:

20 40 60 80 100

a. If the claims are assumed to be uniform, calculate the maximum likelihood estimator of θ.

Solution:

MLE of θ is MAX(20 40 60 80 100) = 100

b. If claims are assumed to be distributed gamma with α = 2, calculate the maximum likelihood estimator of θ.

Solution:

MLE is that $\alpha\theta = \overline{X}$

$$\overline{X} = \frac{20+40+60+80+100}{5} = 60$$

$$\alpha\theta = \overline{X} \text{ ==> } (2)(\theta) = 60 \text{ ==> } \theta = 30$$

13. You are given the following four disability claims which are believed to come from a population which is distributed following a Weibull distribution with $\tau = 2$:

    500, 1200, 2400, 5000

    Estimate θ using the Method of Percentile Matching and the 25th percentile.

    Not An Applicable Question.

14. Using the inversion method of simulation with pseudo random numbers selected on a uniform distribution from 0 to 1, you are to simulate claims for the following three distributions:

    i. **Single Parameter** Pareto distribution with α = 3 and θ = 1000

    ii. $$F(x) = \begin{cases} 0.002x & 0 \le x < 300 \\ \\ 0.002x + 0.1 & 300 \le x \le 450 \end{cases}$$

    iii. $$F(x) = \begin{cases} 0.001x & 0 \le x < 650 \\ 0.650 & 650 \le x \le 850 \\ 0.002x - 1.05 & 850 < x \le 1025 \end{cases}$$

    Complete the following table (Show your work!)

u **	**x** for Distribution i**	**x** for Distribution ii**	**x** for Distribution iii**
0.30			
0.65			
0.90			

Not An Applicable Question.

15. During the last 30 day month, the following data on the number of automobile accidents on the Purdue campus has been collected:

Number of Accidents	**Number of Days**
0	8
1	12
2	5
3	4
4	0
5	1

The Police Department believes that the number of accidents in a day is distributed as a Poisson distribution.

What is the 90% linear confidence interval for $\hat{\lambda}$?

Solution:

$$\hat{\lambda} = \overline{X} = \frac{(0)(8)+(1)(12)+(2)(5)+(3)(4)+(4)(0)+(5)(1)}{30} = 1.3$$

$$Var[\hat{\lambda}] = \frac{\hat{\lambda}}{n} = \frac{1.3}{30} = 0.04333333$$

$$\text{90\% Confidence Interval} \Rightarrow 1.30 \pm 1.645\sqrt{0.04333333} = (0.95757;1.64243)$$