## STAT 47900

Test 3

## Spring 2017

May 3, 2017

1. Brandon has the following sample of five claims:

$$
\begin{array}{lllll}
20 & 35 & 45 & 55 & 70
\end{array}
$$

Brandon wants to develop a continuous distribution of the amount of claims using the Kernel Density Model using the uniform kernel with a span of 30 .

Calculate $\hat{F}(50)$.
2.
a. Calculate the mean and variance for the distribution in Question 1.
b. Determine the mean and variance of the resulting distribution if Brandon had used the triangular kernel with a span of 30 with the data in Question 1.
3. A health insurance company has the following sample of five claims:

500, 500, 700, 800, 1000

The company wants to model their claims using a Gamma distribution.

Calculate the parameters for the Gamma distribution using the Method of Moments.
4. The variance of a distribution will be estimated using:

$$
\hat{\sigma}^{2}=\frac{\sum(X-\bar{X})^{2}}{n}
$$

The following sample is selected from the distribution:

$$
5,5,8
$$

Using the bootstrap method, estimate the mean square error in the above estimator.
5. During the last calendar year, Purdue incurred 100 worker's compensation claims which are summarized in the following table:

| Amount of Claim | Number of Claims |
| :---: | :---: |
| $0-5000$ | 13 |
| $5000-15,000$ | 17 |
| $15,000-45,000$ | 38 |
| More than 45,000 | 32 |

Zihe wants to test the following hypothesis using the Chi Square Test at a $10 \%$ significance level:
$\mathrm{H}_{0}$ : Workers Compensation claims are uniformly distributed with $\theta=60,000$.
$H_{1}$ : Workers Compensation claims are not uniformly distributed with $\theta=60,000$.

Calculate the Chi Square test statistic.

Determine the critical value for this test.

State your conclusion with regard to the hypothesis.
6. Using simulation, Tracy estimates the claim payments that will be made under a dental insurance policy. The policy has a $\$ 50$ deductible for each claim. The maximum that will be paid under the dental policy for all claims in a year is $\$ 500$.

You are given that the number of claims is distributed as a Poisson distribution with $\lambda=2$.

The amount of each claim is distributed as an exponential distribution with $\theta=200$.

Using simulation, Tracy wants to estimate the total claims that will need to be paid under the dental policy. She does so by estimating the claims for each insured. Dali and Miao are the first two insureds. First, Tracy determines the number of claims for Dali and then the amount of each claim for Dali. Next, Tracy determines the number of claims for Miao. Finally, Tracy simulates the amount of each of Miao's claims.

The random numbers used in the simulation are:
$\begin{array}{lllllllllllll}0.20 & 0.55 & 0.85 & 0.50 & 0.30 & 0.95 & 0.45 & 0.60 & 0.95 & 0.75 & 0.05\end{array}$

Calculate the amount paid for Dali and the amount paid for Miao.
7. Ye Fire Company has experienced the following four claims this year:

5000 25,000 50,000 60,000

The company's actuary, Tony, wants to test the following hypothesis using the KolmogorovSmirnov Test with a $5 \%$ significance level:
$H_{0}$ : Claims are distributed as an exponential distribution with $\theta=40,000$.
$H_{1}$ : Claims are not distributed as an exponential distribution with $\theta=40,000$.

Calculate the Kolmogorov-Smirnov test statistic.

Calculate the critical value.

State your conclusion with regard to Tony's hypothesis.
8. The following ten dental claim payments were made by Dowell Dental:
$30,50,60,100,100,120,140,150,150,150$

Claims under Dowell Dental are subject to an upper limit of 150 .
Dowell models claims using an Exponential distribution.

Determine the Maximum Likelihood Estimator for $\theta$.
9. Combs Dental Insurance Company has the following data from 1000 insureds:

| Number of Claims in One Year | Number of Insureds |
| :---: | :---: |
| 1 | 100 |
| 2 | 400 |
| 3 | 250 |
| 4 | 150 |
| 5 | 100 |

Combs believes that the number of claims in one year is distributed as a binomial distribution.

Use the method of moments to estimate the parameters for a binomial distribution.
10.
a. State the principle of parsimony.
b. Which score based approach to determining a model is consistent with the principle of parsimony?
11. Chen Automobile Insurance Company has the following information regarding three auto claims:
a. One claim is equal to 5000 .
b. Two claims exceed 5000 .

The claims are assumed to follow a Pareto distribution which $\alpha=3$.

Calculate the Maximum Likelihood Estimator for $\theta$.
12. You are given the following sample of five claim amounts:

20406080100
a. If the claims are assumed to be uniform, calculate the maximum likelihood estimator of $\theta$.
b. If claims are assumed to be distributed gamma with $\alpha=2$, calculate the maximum likelihood estimator of $\theta$.
13. You are given the following four disability claims which are believed to come from a population which is distributed following a Weibull distribution with $\tau=2$ :

500, 1200, 2400, 5000

Estimate $\theta$ using the Method of Percentile Matching and the $25^{\text {th }}$ percentile.
14. Using the inversion method of simulation with pseudo random numbers selected on a uniform distribution from 0 to 1 , you are to simulate claims for the following three distributions:
i. Single Parameter Pareto distribution with $\alpha=3$ and $\theta=1000$
ii. $\quad F(x)= \begin{cases}0.002 x & 0 \leq x<300 \\ 0.002 x+0.1 & 300 \leq x \leq 450\end{cases}$
iii. $\quad F(x)=\left\{\begin{array}{lr}0.001 x & 0 \leq x<650 \\ 0.650 & 650 \leq x \leq 850 \\ 0.002 x-1.05 & 850<x \leq 1025\end{array}\right.$

Complete the following table (Show your work!)

| $\mathbf{u} * *$ | $\mathbf{x}^{* *}$ for Distribution $\mathbf{i}$ | $\mathbf{x}^{* *}$ for Distribution ii | $\mathbf{x}^{* *}$ for Distribution iii |
| :---: | :---: | :---: | :---: |
| 0.30 |  |  |  |
| 0.65 |  |  |  |
| 0.90 |  |  |  |

15. During the last 30 day month, the following data on the number of automobile accidents on the Purdue campus has been collected:

| Number of Accidents | Number of Days |
| :---: | :---: |
| 0 | 8 |
| 1 | 12 |
| 2 | 5 |
| 3 | 4 |
| 4 | 0 |
| 5 | 1 |

The Police Department believes that the number of accidents in a day is distributed as a Poisson distribution.

What is the $90 \%$ linear confidence interval for $\hat{\lambda}$ ?

