STAT 479
Quiz 2
Spring 2020

February 4, 2020

1. The Bell Casualty Company sells automobile coverage. The coverage has an ordinary deductible of 2500 and an upper limit of 20,000.

   The losses under this automobile insurance are distributed as a Pareto distribution with $\theta = 60,000$ and $\alpha = 5$.

   Calculate $E[Y^P]$.

   Solution:

$$E[Y^P] = \frac{E[X \wedge 20,000] - E[X \wedge 2500]}{1 - F(2500)}$$

$$= \frac{\left(\frac{\theta}{\alpha - 1}\right)\left[1 - \left(\frac{\theta}{20,000 + \theta}\right)^{\alpha - 1}\right] - \left(\frac{\theta}{\alpha - 1}\right)\left[1 - \left(\frac{\theta}{2500 + \theta}\right)^{\alpha - 1}\right]}{\left(\frac{\theta}{2500 + \theta}\right)^{\alpha}} =$$

$$\frac{\left(\frac{60,000}{5 - 1}\right)\left[1 - \left(\frac{60,000}{20,000 + 60,000}\right)^{5 - 1}\right] - \left(\frac{60,000}{5 - 1}\right)\left[1 - \left(\frac{60,000}{2500 + 60,000}\right)^{5 - 1}\right]}{\left(\frac{60,000}{2500 + 60,000}\right)^{5}}$$

$$= 9804.23$$

2. Let N be the random variable which represents the number of students who utilize the elevators in the Math Building in an hour. The distribution of N is modeled as a zero-modified Poisson distribution with $\lambda = 4$ and $p_0^M = 0.4$.

   Calculate the $E[N]$ and the $Var[N]$.

   Solution:

   $E[N] = (1 - p_0^M) E[N]$ for a zero truncated distribution

   $$= (1-0.4)\left(\frac{\lambda}{1-e^{-\lambda}}\right) = (0.6)\left(\frac{4}{1-e^{-4}}\right) = (0.6)(4.074629441) = 2.4448$$

   $Var[N] = (1 - p_0^M) Var[N]$ for a zero truncated distribution

   $+ (1 - p_0^M)(p_0^M)\left[E(N)\right]^2$ for a zero truncated distribution

   $$(1-0.4)\left(\frac{\lambda[1-(\lambda+1)e^{-\lambda}}{(1-e^{-\lambda})^2}\right) + (1-0.4)(0.4)\left(\frac{\lambda}{1-e^{-\lambda}}\right)^2$$

   $$= (0.6)\left(\frac{4[1-(4+1)e^{-4}}{(1-e^{-4})^2}\right) + (0.6)(0.4)(4.074629441)^2 = 6.1656$$