1. (10 points) You are given the following:

   i. The random variable X has the density function $f(x) = \{2(\theta - x)\}/\theta^2$, $0 < x < \theta$

   ii. A random sample of two observations of X yields values of 0.50 and 1.00.

   Determine the maximum likelihood estimate for θ.

   Solution:

$$L(\theta) = f(0.5)\,f(1.0) = \left(\frac{2(\theta-0.5)}{\theta^2}\right)\left(\frac{2(\theta-1)}{\theta^2}\right) = 4(\theta-0.5)(\theta-1)\theta^{-4}$$

$$l(\theta) = \ln(4) + \ln(\theta-0.5) + \ln(\theta-1) - 4\ln\theta$$

$$l'(\theta) = \frac{1}{(\theta-0.5)} + \frac{1}{(\theta-1)} - \frac{4}{\theta} = 0$$

$$\theta(\theta-1) + \theta(\theta-0.5) - 4(\theta-0.5)(\theta-1) = 0$$

$$\theta^2 - \theta + \theta^2 - 0.5\theta - 4\theta^2 + 6\theta - 2 = 0 \Longrightarrow -2\theta^2 + 4.5 - 2 = 0$$

$$\theta = \frac{-4.5 \pm \sqrt{(4.5)^2 - (4)(-2)(-2)}}{-4} = 0.6096 \quad or \quad 1.64$$

But $\theta = 1.64$ since we know $0 < x < \theta$ and one x is 1.0.

2. I would like to receive 10 points for this question.

   True or False