STAT 479
Quiz 5
Spring 2020

April 7, 2020

1. You are given that aggregate losses for various risks a portfolio have the following probability distributions:

	Number of Risks	Probability of a Loss	Loss Amount	Probability of Loss	Loss Amount	Probability of Loss
Risk 1	60	0.2	100	0.9	500	0.1
Risk 2	30	0.5	200	0.8	800	0.2
Risk 3	10	0.6	300	0.7	1000	0.3

a. (1 point) Calculate the EPV for the claim frequency.

Solution:

	Number	Prob of Claim	E(N)	Var(N)
Risk 1	60	0.2	0.2	(0.2)(0.8) = 0.16
Risk 2	30	0.5	0.5	(0.5)(0.5) =0.25
Risk 3	10	0.6	0.6	(0.6)(0.4) = 0.24

$$EPV = (0.6)(0.16) + (0.3)(0.25) + (0.1)(0.24) = 0.195$$

b. (1 point) Calculate the VHM for the claim frequency.

Solution:

$$E[N] = (0.6)(0.2) + (0.3)(0.5) + (0.1)(0.6) = 0.33$$

$$E[N^2] = (0.6)(0.2)^2 + (0.3)(0.5)^2 + (0.1)(0.6)^2 = 0.135$$

$$VHM = E[N^2] - \left(E[N]\right)^2 = 0.135 - (0.33)^2 = 0.0261$$

c. (3 points) A risk is chosen at random and observed for three years. During the three years, the insured has 2 claims. Use Buhlmann Credibility to estimate the expected claim frequency for this insured for the fourth year.

Solution:

$$Z = \frac{N}{N+K} \text{ ==> } K = \frac{EPV}{VHM} = \frac{0.195}{0.0261} = 7.47126 \text{ ==> } Z = \frac{3}{3+7.47126} = 0.28650$$

$N = 3$ because we observed 3 years.

$$\text{Expected Frequency in 4th year} = (Z)(\text{observed frequency}) + (1-Z)(\text{A Priori Expectation})$$

$$=(0.28650)\left(\frac{2}{3}\right) + (1-0.28650)(0.33) = 0.42646$$

d. (2 points) Calculate the EPV for the claim severity.

Solution:

$$E[X \mid 1] = (0.9)(100) + (0.1)(500) = 140$$
$$E[X^2 \mid 1] = (0.9)(100)^2 + (0.1)(500)^2 = 34,000$$
$$Var[X \mid 1] = 34,000 - (140)^2 = 14,400$$

$$E[X \mid 2] = (0.8)(200) + (0.2)(800) = 320$$
$$E[X^2 \mid 2] = (0.8)(200)^2 + (0.2)(800)^2 = 160,000$$
$$Var[X \mid 2] = 160,000 - (320)^2 = 57,600$$

$$E[X \mid 3] = (0.7)(300) + (0.3)(1000) = 510$$
$$E[X^2 \mid 3] = (0.7)(300)^2 + (0.3)(1000)^2 = 363,000$$
$$Var[X \mid 3] = 363,000 - (510)^2 = 102,900$$

$$\text{Pr(Type 1 and Claim)} = (0.6)(0.2) = 0.12$$
$$\text{Pr(Type 2 and Claim)} = (0.3)(0.5) = 0.15$$
$$\text{Pr(Type 3 and Claim)} = (0.1)(0.6) = 0.06$$
Total Prob = 0.12 + 0.15 + 0.06 = 0.33

$$EPV = \left(\frac{0.12}{0.33}\right)(14,400) + \left(\frac{0.15}{0.33}\right)(57,600) + \left(\frac{0.06}{0.33}\right)(102,900) = 50,127.27$$

e. (2 points) Calculate the VHM for the claim severity.

Solution:

$$E[Mean] = \left(\frac{0.12}{0.33}\right)(140) + \left(\frac{0.15}{0.33}\right)(320) + \left(\frac{0.06}{0.33}\right)(510) = 289.09$$

$$E[Mean^2] = \left(\frac{0.12}{0.33}\right)(140)^2 + \left(\frac{0.15}{0.33}\right)(320)^2 + \left(\frac{0.06}{0.33}\right)(510)^2 = 100,963.64$$

$$VHM = 100,963.64 - (289.09)^2 = 17,390.61$$

f. (5 points)A risk is chosen at random and observed for three years. During the three years, the insured has 2 claims for a total of 1000. Use Buhlmann Credibility to estimate the expected claim severity for this insured for the fourth year.

Solution:

$$Z = \frac{N}{N+K} \text{ ==> } K = \frac{EPV}{VHM} = \frac{50{,}127.27}{17{,}390.61} = 2.8824 \text{ ==> } Z = \frac{2}{2+2.8824} = 0.40963$$

$N = 2$ because we observed 2 claims.

$$\text{Expected Frequency in 4th year} = (Z)(\text{observed severity}) + (1-Z)(\text{A Priori Expectation})$$

$$=(0.40963)(500) + (1-0.40963)(289.09) = 375.49$$

g. (2 points) Calculate the EPV for the pure premium.

Solution:

$$E[PP|1] = E[N|1]E[X|1] = (0.2)(140) = 28$$
$$Var[PP|1] = E[N|1]Var[X|1] + \left(E[X|1]\right)^2 Var[N|1] = (0.2)(14,400) + (140)^2(0.16) = 6016$$

$$E[PP|2] = E[N|2]E[X|2] = (0.5)(320) = 160$$
$$Var[PP|2] = E[N|2]Var[X|2] + \left(E[X|2]\right)^2 Var[N|2] = (0.5)(57,600) + (320)^2(0.25) = 54,400$$

$$E[PP|3] = E[N|3]E[X|3] = (0.6)(510) = 306$$
$$Var[PP|3] = E[N|3]Var[X|3] + \left(E[X|3]\right)^2 Var[N|3] = (0.6)(102,900) + (510)^2(0.24) = 124{,}164$$

$$EPV = (0.6)(6016) + (0.3)(54,400) + (0.1)(124{,}164) = 32,346$$

h. (1 point) Calculate the VHM for the pure premium.

Solution:

$$E[PP] = (0.6)(28) + (0.3)(160) + (0.1)(306) = 95.4$$

$$E[PP^2] = (0.6)(28)^2 + (0.3)(160)^2 + (0.1)(306)^2 = 17,514$$

$$VHM = 17,514 - (95.4)^2 = 8412.84$$

i. (3 points) A risk is chosen at random and observed for three years. During the three years, the insured has 2 claims for a total loss of 1000. Use Buhlmann Credibility to estimate the expected pure premium for this insured for the fourth year.

Solution:

$$Z = \frac{N}{N+K} \text{ ==> } K = \frac{EPV}{VHM} = \frac{32,346}{8412.84} = 3.8448 \text{ ==> } Z = \frac{3}{3+3.8448} = 0.4383$$

$N = 3$ because we observed 3 years.

$$\text{Expected Pure Premium in 4th year} = (Z)(\text{observed severity}) + (1-Z)(\text{A Priori Expectation})$$

$$=(0.4383)\left(\frac{1000}{3}\right) + (1-0.4383)(95.4) = 199.69$$