STAT 479
Test1
Spring 2020

February 13, 2020

1. Let X be the amount of a claim. Assume that X is distributed uniformly between 1000 and 2500.

    a. Calculate the $E[X]$.

    Solution:

    $$E[X] = \frac{a+b}{2} = \frac{1000+2500}{2} = 1750$$

    b. Calculate the $Var[X]$.

    Solution:

    $$Var[X] = \frac{(b-a)^2}{12} = \frac{(2500-1000)^2}{12} = 187,500$$

    c. Calculate $TVaR_{0.8}(X)$.

    Solution:

    $$TVaR_p = \frac{\pi_p + b}{2} = \frac{2200+2500}{2} = 2350$$

    $$\pi_p = a + (p)(b-a) = 1000 + (0.8)(2500-1000) = 2200$$

2. The Niu Insurance Company sells hospital indemnity insurance. Let the random variable N represent the number of claims each year for each insured. Let the random variable X represent the amount of each claim. Let the random variable S represent the aggregate claims per insured in a year.

   The number of claims per insured in a year is distributed as a negative binomial with parameters of $\gamma = 5$ and $\beta = 0.2$.

   The amount of each claim is distributed as a Pareto distribution with $\theta = 5000$ and $\alpha = 5$.

   The amount of each claim and the number of claims is independent.

   a. Calculate the $E[S]$.

      Solution:

      $$E[S] = E[N]E[X] = [(5)(0.2)]\left[\frac{5000}{5-1}\right] = 1250$$

   b. Calculate the $Var[S]$

      Solution:

      $$Var[S] = E[N]Var[X] + Var[N]\left(E[X]\right)^2$$

      $$E[N] = 1 \qquad Var[X] = \frac{\theta^2\alpha}{(\alpha-1)^2(\alpha-2)} = \frac{(5000)^2(5)}{(4)^2(3)} = 2,604,166.67$$

      $$Var[N] = (5)(0.2)(1.2) = 1.2 \qquad E[X] = 1250$$

      $$Var[S] = (1)(2,604,166.67) + (1.2)(1250)^2 = 4,479,166.67$$

   c. Niu sells 4000 polices to 4000 independent insured. Assuming the normal distribution, calculate the probability that the aggregate claims will exceed 5,100,000.

      Solution:

      $$E[Port] = (4000)(1250) = 5,000,000 \qquad Var[Port] = (4000)(4,479,166.67)$$

      $$Z = \frac{5,100,000 - 5,000,000}{\sqrt{(4000)(4,479,166.67)}} = 0.747 \quad \text{Answer} = 1 - \Phi(0.75) = 1 - 0.7734 = 0.2266$$

3. Dental claims are distributed exponentially with θ = 100. You want to discretize the claims using a span of 20 using both the Method of Rounding and the Method of Moment Matching where you will match the mean.

   Calculate the probability assigned to the range that includes 100 under each method. Please provide your answer to at least five decimal places.

   Solution:

   Method of Rounding

$$f_5 = F(110) - F(90) = \left[1 - e^{-110/100}\right] - \left[1 - e^{-90/100}\right] = 0.073699$$

   Method of Moment Matching

$$f_5 = \frac{2E[X \wedge 100] - E[X \wedge 120] - E[X \wedge 80]}{20} = 0.073821$$

4. Losses are modeled assuming that the amount of all losses is 40 and that the number of losses follows a geometric distribution with a mean of 4.

   Calculate the net stop loss premium for coverage with an aggregate deductible of 110.

   Solution:

$$E[(S-d)_+] = E[S] - E(S \wedge d)$$

$$E[S] = E[N]E[X] = (4)(40) = 160$$

$$E[S \wedge 110] = (0)(p_0) + (40)(p_1) + 80(p_2) + 110(1 - p_0 - p_1 - p_2)$$

$$= (0)\left[\frac{1}{5}\right] + (40)\left[\frac{4}{5^2}\right] + (80)\left[\frac{4^2}{5^3}\right] + 110\left[1 - \frac{1}{5} - \frac{4}{5^2} - \frac{4^2}{5^3}\right] = 72.96$$

$$\text{Net Stop Loss Premium} = 160 - 72.96 = 87.04$$

5. Homeowners Liability claims are distributed as a two point mixture distribution. Each distribution has an equal weight.

   Distribution 1 is an exponential distribution with $\theta = 100,000$.

   Distribution 2 is a Pareto distribution with $\alpha = 3$ and $\theta = 50,000$

   Calculate the $\sqrt{Var[X]}$.

   Solution:

$$E[X] = 0.5(100,000) + 0.5\left(\frac{50,000}{2}\right) = 62,500$$

$$E[X^2] = 0.5\left((100,000)^2(2)\right) + (0.5)\left(\frac{(50,000)^2(2)}{(2)(1)}\right) = 11,250,000,000$$

$$\sqrt{Var[X]} = \sqrt{11,250,000,000 - (62,500)^2} = 85,695.68$$

6. Losses for travel accident insurance have an exponential distribution with a mean of 800. Let the random variable X represent the amount of the loss.

   During 2019, Yee Insurance Company writes travel accident insurance with an upper limit of 1000 and a franchise deductible of 100.

   a. Calculate the expected value of Y^L which is the payment per loss. The answer is between 560 and 570. If you do not get this answer, use 560 for part b.

      Solution:

      $$E[Y^L] = E[X \wedge 1000] - E[X \wedge 100] + d[1 - F(100)]$$

      $$= \theta(1 - e^{-1000/\theta}) - \theta(1 - e^{-100/\theta}) + (100)\left[1 - (1 - e^{-100/\theta})\right]$$

      $$= 800(1 - e^{-1000/800}) - 800(1 - e^{-100/800}) + (100)\left[e^{-100/800})\right] = 565.04$$

   During 2020, claims are expected to subject to uniform inflation of 10%. Yee decides to sell insurance in 2020 with no upper limit and an ordinary deductible of d. The ordinary deductible is established so the expected payment per loss for this new policy in 2020 is the same as for 2019 policy with the franchise deductible and upper limit.

   b. Determine d.

      Solution:

      $$\theta_{2020} = (\theta_{2019})(1.1) = 880$$

      $$E[Y^L_{2020}] = E[X] - E[X \wedge d] = 565.04$$

      $$\theta - \theta(1 - e^{-d/\theta}) = 565.04 ==> e^{-d/880} = \frac{565.04}{880}$$

      $$d = (880)\left[-\ln\left(\frac{565.04}{880}\right)\right] = 389.86$$

7. You are given that N is distributed as a zero modified Poisson. You are also given that:

   a. $p_2^M = 0.24168$ and

   b. $p_3^M = 0.18529$

   Calculate p_0^M .

   Solution:

$$p_3^M = \left(a + \frac{b}{k}\right) p_2^M$$

For Poisson, $a = 0$ and $b = \lambda$ ==> $0.18529 = \left(\frac{\lambda}{3}\right) 0.24168$ ==> $\lambda = 2.3$

$$p_2^M = (1 - p_0^M) p_2^T \Longrightarrow 0.24168 = (1 - p_0^M)\left[\frac{\lambda^2}{2!(e^{\lambda} - 1)}\right] = (1 - p_0^M)\left[\frac{(2.3)^2}{2(e^{2.3} - 1)}\right]$$

$$p_0^M = 1 - (0.24168)\left[\frac{2(e^{2.3} - 1)}{(2.3)^2}\right] = 0.18$$

8. The Farber Game Company has a dental insurance policy, the number of dental claims in a year is distributed as a follows:

Number of Claims	Probability
0	0.05
1	0.25
2	0.60
3	0.08
4	0.02

The amount of a claim under this policy is distributed as follows:

Amount of Claims	Probability
25	0.25
50	0.50
100	0.15
500	0.05
1000	0.05

The amount of each claim and the number of claims is independent.

Calculate $f_S(100)$, the probability that aggregate claims under a policy will be 100 during a one year period.

Solution:

$f(100)$ ==> One claim of 100 or two claims of 50
or three claims of 25, 25, and 50 in any order or 4 claims of 25

$$f(100) = (0.25)(0.15) + (0.60)(0.5)^2 + (0.08)(3)(0.25)^2(0.5) + (0.02)(0.25)^4$$

$$= 0.19508$$

9. You are given $F(x) = \frac{x^2}{100}$ for 0 < x < 10.

a. Calculate the $E[X]$.

Solution:

$$f(x) = \frac{x}{50} \Longrightarrow E[X] = \int_0^{10} x \cdot f(x) \cdot dx = \int_0^{10} \frac{x^2}{50} \cdot dx = \frac{x^3}{150}\Bigg]_0^{10} = \frac{1000}{150} = 6.6666666666$$

b. Calculate the $Var[X]$.

Solution:

$$Var[X] = E[X^2] - \left(E[X]\right)^2 = 50 - (6.666666666)^2 = 5.55555555$$

$$E[X^2] = \int_0^{10} x^2 \cdot f(x) \cdot dx = \int_0^{10} \frac{x^3}{50} \cdot dx = \frac{x^4}{200}\Bigg]_0^{10} = \frac{10{,}000}{200} = 50$$

c. Calculate the median.

Solution:

$$Median = \pi_{0.5} \text{ such that } F(\pi_{0.5}) = 0.5$$

$$F(\pi_{0.5}) = \frac{(\pi_{0.5})^2}{100} = 0.5 \Longrightarrow \pi_{0.5} = \sqrt{50} = 7.0711$$

d. Calculate $TVaR_{0.90}(X)$.

Solution:

$$TVaR_{0.90}(X) = \frac{\int_{\pi_{0.9}}^{10} x \cdot f(x) \cdot dx}{1 - F(\pi_{0.9})} = \frac{\int_{9.4868}^{10} \frac{x^2}{50} \cdot dx}{1 - 0.9} = 10\left[\frac{x^3}{150}\right]_{9.4868}^{10} = 9.7457$$

$$F(\pi_{0.9}) = \frac{x^2}{100} = 0.9 \Longrightarrow \pi_{0.9} = \sqrt{90} = 9.4868$$