STAT 479
Test 3
Spring 2020

May 9, 2020

Introduction

Andrew Auto Insurance Company provides automobile insurance. Andrew offers collision, comprehensive, liability, and uninsured/underinsured driver coverages.

Andrew splits drivers into three categories – Safe, Not So Safe, and Reckless. You have the following information for **collision** coverage:

Type	Proportion of Total Drivers	Claim Frequency Poisson Annual	Severity Gamma
Safe	0.5	0.04	$\alpha=3, \theta=1000$
Not So Safe	0.3	0.10	$\alpha=4, \theta=1000$
Reckless	0.2	0.25	$\alpha=5, \theta=1000$

You have the following information for **uninsured/under insured** coverage:

Type	Proportion of Total Drivers	Chance of Claim
Safe	0.5	0.10
Not So Safe	0.3	0.20
Reckless	0.2	0.35

Under **comprehensive** coverage, the probability of exactly one claim in a year for one insured is θ. The probability of zero claims in a year is $(1-\theta)$. The probability of a claim, θ, varies within the population and is based on a uniform distribution from 0.2 to 1.

Under **comprehensive** coverage, individual losses on a policy are distributed as a Pareto with $\alpha=4$ and θ. The parameter θ is uniformly distributed between 2000 and 5200.

For **liability** coverage, the number of claims that a particular insured makes in a year follows a Poisson distribution with a mean of λ. The value of λ for the population of insureds follows a Gamma distribution with $\alpha=4$ and $\theta=\frac{1}{20}$.

For each driver, frequency and severity are independent.

1. For each of the following situations, state which of the four coverage (collision, comprehensive, liability, and uninsured/underinsured driver) would pay. More than one coverage may pay.
    a. Molly is driving on the interstate. Molly lives in a fault state. A tire of a truck in front of her throws a rock that hits and cracks her windshield. Which coverage(s) under Molly's policy will pay to fix Molly's windshield?
    b. As Molly gets off the interstate, she runs into the back of Zane's car as she could not see out her cracked windshield. This causes damage to both Molly's car and Zane's car. Zane ends up going the hospital with a broken leg. Zane is not insured.
        i. Which coverage(s) under Molly's policy will pay to fix Molly's car (other than the windshield)?
        ii. Which coverage(s) under Molly's policy will pay to fix Zane's car?
        iii. Which coverage(s) under Molly's policy will pay for Zane's hospital visit?

2. What is the expected value of the process variance of the claim severities (for the observation of a single claim) under collision coverage?

3. What is the variance of the hypothetical mean severities (for the observation of a single claim) under collision coverage?

4. Over several years, for Natasha, an individual driver under collision coverage, you observe a single claim of 20,000. Use Buhlmann Credibility and the information in Questions 2 and 3 to estimate Natasha's future average claim severity.

5. What is the expected value of the process variance of the pure premium (for the observation of a single exposure) under collision coverage?

6. Under uninsured/under insured coverage, each risk has either zero claims or one claim. The information regarding risks are above in the introduction.

   Jiaxin is selected at random. You observe one claim in a year from uninsured/under insured coverage. Using Bayes Analysis, what is the expected annual claim frequency for Jiaxin during the next year?

7. Shina is selected from the population and observed for three years. Under her comprehensive coverage, she has one claim per year in two of the three years and zero claims in one of the three years.

    a. Calculate the posterior density function of θ for Shina.

       Solution:

    b. Calculate the expected number of claims for Shina for next year.

8. A policyholder, David, is chosen at random from the population of insureds and is observed with regard to his liability insurance. David is observed for two years. In the first year, he has four claims. In the second year, he has one claim.

   Calculate the probability that David will have zero claims in the third year of observation.

9. For a group of policyholders with comprehensive coverage, we observe the following two years of claims experience:

Year	**Number of Losses**	**Total Loss**
1	20	30,000
2	16	17,600

Use Buhlman-Straub Credibility to estimate the size of one claim for this group next year.

10. During 2019, Andrew Auto collects the following liability premium amounts:

Month	**January**	**March**	**May**	**July**
Premium Collected	2,000,000	2,400,000	1,800,000	1,200,000

All premiums are paid on the first day of the month and all premiums are annual premiums.

Madison, the company's actuary, expects a loss ratio of 65%.

During 2019, the company paid losses for liability claims incurred in 2019 of 2,500,000.

Use the loss ratio method to determine the reserve for liability coverage that should be held on December 31, 2019.

11. Jake (not from State Farm) is one of the other actuaries at Andrew Auto. He has the following Paid Claims triangle for collision coverage:

Cumulative Loss Payments by Development Year						
Accident Year	**Development Year**					
	0	1	2	3	4	5
2014	1,000,000	1,500,000	1,700,000	1,800,000	1,850,000	1,875,000
2015	1,100,000	1,750,000	1,775,000	1,825,000	1,870,000	
2016	1,200,000	1,900,000	2,200,000	2,350,000		
2017	1,500,000	2,200,000	2,500,000			
2018	2,000,000	2,900,000				
2019	2,500,000					

There is no further development after year 5.

Calculate the loss reserve on December 31, 2019 using the chain ladder method with arithmetic average loss development factors.

12. Determine the total amount of collision claims paid in 2019.

13. The following table shows the link ratios for cumulative payments for comprehensive coverage based on the chain ladder method:

Development Years	**Link Ratio**
1/0	1.50
2/1	1.20
3/2	1.05

There is no further development after three years.

For accident year 2019, the earned premium was 700,000. The expected loss ratio was 0.80. The claims paid in 2019 totaled 305,000.

For the claims from accident year 2019, determine the reserves as of December 31, 2019 using the Bornhuetter-Ferguson method.

14. Jack is the rate making actuary for Andrew Auto. He is setting rates for the auto coverage which is a short term insurance product. You are given the following data:

Calendar Year	Earned Premium
2017	10,000
2018	12,000
2019	8,000

Assume that all policies are one year policies and the policies are issued uniformly throughout the year.

The following rate changes have occurred:

Date	Rate Change
March 15, 2017	10% Increase
July 30, 2018	8% Increase
October 31, 2019	4% Decrease

Using the parallelogram method, calculate the earned premium for 2017, 2018, and 2019 based on current rates.

15. Beau, one of the rate making actuaries at Andrew Auto, is determining the new average gross premium rate based on the following data:

Expected Effective Period Incurred Losses	12,000,000
Earned Exposure Units	20,000
Earned Premium at Current Rates	18,000,000
Fixed Expenses	120,000
Permissible Loss Ratio	0.6

Determine the new average gross premium rate.

16. For collision coverage for Andrew Auto, these assumptions are repeated from above:

Type	Proportion of Total Drivers	Claim Frequency Poisson Annual	Severity Gamma
Safe	0.5	0.04	$\alpha=3, \theta=1000$
Not So Safe	0.3	0.10	$\alpha=4, \theta=1000$
Reckless	0.2	0.25	$\alpha=5, \theta=1000$

Calculate the indicated differentials for each Type given that Safe is the base rate.

17. During 2019, Andrew Auto experienced the following loss ratios for collision coverage based on the indicated differentials developed in Question 16:

Type	Loss Ratio
Safe	60%
Not So Safe	63%
Reckless	51%

Use the loss ratio method to determine the new indicated differentials for 2020 for the Not So Safe and Reckless categories.

18. For collision coverage, Andrew Auto sells coverage with ordinary deductibles of 250, 500, and 1000. Andrew has the following data:

Loss Size	Number of Losses	Ground Up Total Loss
0 – 250	1000	150,000
251 - 500	2000	650,000
501 – 1000	6000	4,320,000
Over 1000	20,000	90,000,000

The base rate is a deductible of 500.

Calculate the indicated deductible relativity for the deductible of 250 and the deductible of 1000.

19. Andrew Auto sells liability coverage with limits up to 5,000,000. Andrew only wants to cover 1,000,000 of liability so Andrew buys reinsurance from Rangarajan Reinsurance. Rangarajan provides coverage for 80% of losses up to 3,000,000 excess of 1,000,000. Rangarajan also provides coverage of 95% for losses up to 1,000,000 excess of 4,000,000.

    Rangarajan retrocedes part of the reinsurance coverage to The White Reinsurance Company. White provides aggregate stop loss reinsurance for Rangarajan that limits Rangarajan payments to 2,500,000.

    Andrew Auto suffers a loss of 4,600,000. Determine how payments will be split between Andrew, Rangarajan, and White.

20. Andrew Auto Insurance Company writes 20 million of premium in automobile liability. The expected loss ratio on this coverage is 80%.

Andrew offers various limits on the liability coverage. Increased Limit Factors based on industry experience are:

Limit	ILF
250,000	1.00
500,000	1.50
1,000,000	1.75
3,000,000	2.00
5,000,000	2.50

Andrew wants to buy reinsurance from Goh Reinsurance LTD. The coverage will cover 100% of losses in the layer between 500,000 and 3,000,000. Goh calculates the expected losses for this reinsurance layer using the exposure rating approach. Goh adds a 20% charge of the total reinsurance premium for expenses and profit.

Determine the total reinsurance premium that Goh will charge to Andrew for this coverage.