STAT 479
Spring 2022
Quiz 1
January 25, 2022

1. You are given $F(x)=\dfrac{x^2}{100}$ for 0 < *x* < 10.

   Calculate $e(5)$. This is $E[(X-5\,|\,X>5)]$.

   Solution:

$$F(x)=\frac{x^2}{100} \text{ ==> } f(x)=F'(x)=\frac{2x}{100}=\frac{x}{50}$$

$$e(5)=\frac{\int_5^{10}(x-5)f(x)dx}{1-F(5)}=\frac{\int_5^{10}(x-5)\left(\frac{x}{50}\right)dx}{1-\frac{5^2}{100}}=\frac{\left[\frac{x^3}{150}-\frac{5x^2}{100}\right]_5^{10}}{0.75}=\frac{\frac{10^3}{150}-\frac{5(10)^2}{100}-\left\{\frac{5^3}{150}-\frac{5(5)^2}{100}\right\}}{0.75}=2.778$$

Or

$$e(5)=\frac{E[X]-E[X\wedge 5]}{1-F(5)}=\frac{6.66666667-4.58333333}{0.75}=2.778$$

$$E[X]=\int_0^{10}(x)f(x)dx=\int_0^{10}(x)\left(\frac{x}{50}\right)dx=\left[\frac{x^3}{150}\right]_0^{10}=6.66666667$$

$$E[X\wedge 5]=\int_0^{5}(x)f(x)dx+\int_5^{10}(5)f(x)dx=\int_0^{5}(x)f(x)dx+(5)(1-F(5))$$

$$=\int_0^{5}(x)\left(\frac{x}{50}\right)dx+(5)(0.75)=\left[\frac{x^3}{150}\right]_0^{5}+3.75=4.58333333$$

2. Losses represented by the random variable of X are uniformly distributed from 0 to the maximum loss. You are given that $Var[X]=62,208$.

   Calculate $TVaR_{0.75}(X)$.

   Solution:

$$Var[X]=\frac{(b-a)^2}{12}=62,208 ==> (b-0)^2=(12)(62,208) ==> b=864$$

$$TVaR_{0.75}=\frac{b+a+0.75(b-a)}{2}=\frac{864+0.75(864)}{2}=756$$