STAT 479
Spring 2022
Quiz 3
February 22, 2022

1. Let N^L be the random variable representing the number of losses for a dental policy with no deducible. N^L is distributed as a Negative Binomial with $\alpha = 2$ and $\beta = 1$.

   Let N^P be the random variable representing the number of losses for the same dental policy with a deductible of 50.

   Each loss under the dental policy is distributed as a Pareto distribution with $\alpha = 4$ and $\theta = 200$.

   Calculate $Var[N^P]$.

   Solution:

   $N^P \sim$ Negative Binomial with $\gamma = 2$ and $\beta' = v(1)$

   $$v = [1 - F_x(50)] = \left[1 - \left(1 - \left(\frac{200}{200+50}\right)^4\right)\right] = 0.4096$$

   $$\beta' = (0.4096)(1) = 0.4096$$

   $$Var[N^P] = \alpha\beta'(1+\beta') = (2)(0.4096)(1+0.4096) = 1.1547$$

2. Anderson Assurance Association (AAA) has this portfolio of policies. Each policy is independent of the other policies.

   a. 200 insureds who are factory workers. The probability of death for each insured who is a factory worker is 0.08. The amount of death benefit is uniformly distributed between 1000 and 2000.

   b. 100 insureds who are executives. The probability of death for each insured who is an executive is 0.05. The amount of death benefit is 10,000 for all executives.

   Let S be the random variable representing the total losses paid during the next year.

   Calculate $Var[S]$.

   Solution:

   Factory Workers

   $$E[X] = \frac{2000+1000}{2} = 1500 \qquad Var[X] = \frac{(2000-1000)^2}{12} = 83,333.33$$

   Executives

   $E[X] = 10,000$ $\quad Var[X] = 0,$ since the death benefit is the same for everyone.

   $$Var[S] = \sum q \cdot Var[X] + q \cdot (1-q)\left(E[X]\right)^2 =$$

   $$200\left[(0.08)(83,333.33) + (0.08)(1-0.08)(1500)^2\right] + 100\left[(0.05)(0) + (0.05)(1-0.05)(10,000)^2\right]$$

   $$= 509,453,333$$