STAT 479
Spring 2022
Test 1

February 10, 2022

1. (10 points) During 2021, Zhang Dental Company sells a policy with no deductibles and no upper limit. Dental losses follow an exponential distribution with a mean of 75.

   During 2022, dental losses are expected to experience uniform inflation of 40%. Zhang decides to implement an ordinary deductible d on its policy so that the expected amount paid in 2022 will be the same as in 2021.

   Determine d .

   Solution:

   $2021:$

   $$E[X_{2021}] = 75 ==> \theta_{2021} = 75$$

   $2022:$

   $$\theta_{2022} = (1.4)(\theta_{2021}) = (1.4)(75) = 105$$

   $$E[Y^L] = E[X_{2022}] - E[X_{2022} \wedge d] ==> 75 = 105 - [105(1 - e^{-d/105})]$$

   $$105(1 - e^{-d/105}) = 30 ==> e^{-d/105} = 1 - \frac{30}{105} ==> -d/105 = \ln(75/105)$$

   $$d = -105(\ln(75/105)) = 35.33$$

2. (10 points) Losses for an insurance policy covering tornado damage are distributed uniformly between 0 and 10,000.

   Cooley Casualty Insurance Company sells a policy to cover these losses. The policy has a franchise deductible of 1000.

   Calculate $E[Y^P]$ for this policy.

   Solution:

$$E[Y^L] = \int_{1000}^{10,000} x \cdot f(x) \cdot dx = \int_{1000}^{10,000} x \cdot \frac{1}{10,000} \cdot dx = \left[\frac{x^2}{20,000} \right]_{1000}^{10,000} = \frac{(10,000)^2 - (1000)^2}{20,000} = 4950$$

$$E[Y^P] = \frac{E[Y^L]}{1 - F(d)} = \frac{4950}{1 - F(1000)} = \frac{4950}{0.9} = 5500$$

3. (15 points) The Rops Warranty Insurance Company has experienced the following number of claims per day during the last 3 days:

1 2 9

Rops' chief actuary Megan decides to treat this as an empirical distribution.

a. (2 points) Calculate the mean of the empirical distribution.

Solution:

$$E[N]=\frac{1+2+9}{3}=4$$

(4 points) Calculate the variance of the empirical distribution.

Solution:

$$Var[N]=\frac{(1-4)^2+(2-4)^2+(9-4)^2}{3}=\frac{38}{3}$$

Her boss Danielle then suggests that Megan model the number of claims per day as a zero modified geometric distribution with the parameters $\beta=3.2$ and p_0^M determined so the mean matches the mean of the empirical distribution.

b. (4 points) Determine p_0^M under this approach.

Solution:

$$E[N^M]=(1-p^M)E[N^T] ==> 4=(1-p^M)(1+\beta)$$

$$4=(1-p^M)(1+3.2) ==> p^M=0.04762$$

c. (5 points) Determine $Var[N]$ using the zero modified geometric distribution.

Solution:

$$Var[N^M]=(1-p_0^M)Var[N^T]+(p_0^M)(1-p_0^M)E[N^T]$$

$$=(1-0.04762)(3.2)(4.2)+(0.04762)(1-0.04762)(4.2)^2$$

$$=13.6$$

4. (15 points) You are given that $\begin{cases} F(x)=0, \ for \ x<0 \\ F(x)=0.5\left(\frac{x}{10}\right)+0.5\left(\frac{x^2}{100}\right), \ for \ 0\le x\le 10 \\ F(x)=1, \ for \ x>10 \end{cases}$.

a. (5 points) Calculate $E[X]$.

Solution:

$$F(x)=0.5\left(\frac{x}{10}\right)+0.5\left(\frac{x^2}{100}\right) ==> f(x)=0.5\left(\frac{1}{10}\right)-0.5\left(\frac{2x}{100}\right)=0.05+0.01x$$

$$E[X]=\int_0^{10} x\cdot f(x)\cdot dx=\int_0^{10} x\cdot(0.05+0.01x)\cdot dx=\left[0.025x^2+\frac{0.01}{3}x^3\right]_0^{10}=5.833333$$

b. (5 points) Calculate $\sqrt{Var[X]}$.

Solution:

$$E[X^2]=\int_0^{10} x^2\cdot f(x)\cdot dx=\int_0^{10} x^2\cdot(0.05+0.01x)\cdot dx=\left[\frac{0.05}{3}x^3+\frac{0.01}{4}x^4\right]_0^{10}=41.666667$$

$$Var[X]=E[X^2]-(E[X])^2=41.666667-(5.833333)^2=7.6388889$$

c. (5 points) Calculate p such that $\pi_p=E[X]+\sqrt{Var[X]}$.

Solution:

$$\pi_p=5.833333+\sqrt{7.6388889}=8.5972$$

$$p=F(8.5972)=0.5\left(\frac{8.5972}{10}\right)+0.5\left(\frac{(8.5972)^2}{100}\right)=0.799$$

5. (10 points) The random variable N represents the number of automobile accidents during any 24 hour period in West Lafayette. N is distributed as a Poisson distribution with a mean of 4.

Calculate the probability that the number of accidents in a 24 hour period will be less than the mode of this distribution. (If there are multiple modes, then you should calculate the probability that N will be less than the smallest mode.)

Solution:

$$p_0 = e^{-\lambda} = e^{-4} = 0.0183156$$

$$p_1 = \lambda e^{-\lambda} = 4e^{-4} = 0.0732626$$

$$p_2 = \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{4^2 e^{-4}}{2} = 0.1465251$$

$$p_3 = \frac{\lambda^3 e^{-\lambda}}{3!} = \frac{4^3 e^{-4}}{6} = 0.1953668$$

$$p_3 = \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{4^4 e^{-4}}{24} = 0.1953668$$

All future values will be less so the mode is 3 and 4.

$$\Pr(N < 3) = p_0 + p_1 + p_2 = 0.0183156 + 0.0732626 + 0.1465251 = 0.23810$$

6. (10 points) Hwang Insurance Company sells Hospital Indemnity policies.

For each insured, the number of claims in a year are distributed as a Geometric distribution with a mean of 0.5.

For each claim, the amount of the claim is distributed as a Gamma distribution with $\alpha = 3$ and $\theta = 1200$.

Hwang Insurance Company has 2500 hospital indemnity policies.

Assuming a normal distribution, calculate the probability that total aggregate claims in a year will be less 4,400,000.

Solution:

For a Geometric:

$$E[X] = 0.5 = \beta$$
$$Var[X] = (\beta)(1+\beta) = (0.5)(1.5) = 0.75$$

For Gamma Distribution:

$$E[X] = \alpha\theta = (3)(1200) = 3600$$
$$Var[X] = \alpha\theta^2 = (3)(1200)^2 = 4,320,000$$

$$E[S] = E[N] \cdot E[S] = (0.5)(3600) = 1800$$

$$Var[X] = E[N] \cdot Var[S] + Var[N] \cdot (E[X])^2 = (0.5)(4,320,000) + (0.75)(3600)^2 = 11,880,000$$

$$E[Port] = (2500)(1800) = 4,500,000$$

$$Var[Port] = (2500)(11,880,000)$$

$$\Pr(Z < 4,400,000) = \Pr\left(Z < \frac{4,400,000 - 4,500,000}{\sqrt{(2500)(11,880,000)}}\right) = \Pr(Z < -0.58026) =$$

$$1 - \phi(0.58) = 1 - 0.7190 = 0.2810$$

7. (10 points) The random variable X is the loss under a medical insurance policy and is distributed as a 2 point mixed distribution. The 2 point mixed distribution is a combination of a gamma distribution with a weight of 0.4 and a Pareto distribution with a weight of 0.6.

   The parameters for the gamma distribution are $\alpha = 2$ and $\theta = 100$.

   The parameters for the Pareto distribution are $\alpha = 5$ and $\theta = 5000$.

   Calculate the standard deviation of X.

   Solution:

   Gamma

   $E[X] = \alpha\theta = (2)(100) = 200$

   $E[X^2] = \theta^2(\alpha+1)(\alpha) = (100)^2(3)(2) = 60,000$

   Pareto

   $$E[X] = \frac{\theta}{\alpha-1} = \frac{5000}{5-1} = 1250$$

   $$E[X^2] = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = \frac{2(5000)^2}{(5-1)(5-2)} = \frac{12,500,000}{3}$$

   Mixture

   $E[X] = 0.4(200) + 0.6(1250) = 830$

   $$E[X^2] = 0.4(60,000) + 0.6\left(\frac{12,500,000}{3}\right) = 2,524,000$$

   $Var[X] = E[X^2] - (E[X])^2 = 2,524,000 - (830)^2 = 1,835,100$

   $\sigma = \sqrt{Var[X]} = \sqrt{1,835,100} = 1354.66$

8. (10 points) Stowe Indemnity Incorporated provides warranty insurance on iPhones. During each year, the aggregate claims are distributed as follows:

Aggregate Amount of Claim	Probability
0	0.1
50	0.2
100	0.25
200	0.30
400	0.08
1000	0.07

a. (3 points) Calculate $E[S]$.

Solution:

$$E[S] = (0)(0.1) + (50)(0.2) + (100)(0.25) + (200)(0.30) + (400)(0.08) + (1000)(0.07)$$

$$= 197$$

Stowe Indemnity purchases stop loss coverage from Schaeffer Stop Loss Company. The stop loss coverage will cover aggregate claims in excess of 250.

b. (7 points) Calculate the net stop loss premium.

Solution:

The easy way:

$$E[(S-250)_+] = (400-250)(0.08) + (1000-250)(0.07) = 64.5$$

Another way:

$$E[S] - E[S \wedge 250] = 197 - 132.5 = 64.5$$

$$E[S \wedge 250] = (0)(0.1) + (50)(0.2) + (100)(0.25) + (200)(0.30) + (250)(0.08) + (250)(0.07)$$

$$= 132.5$$

9. (10 points) For a dental insurance policy, the number of dental claims in a year is distributed as a follows:

Number of Claims	**Probability**
0	0.05
1	0.25
2	0.60
3	0.08
4	0.02

The amount of a claim under this policy is distributed as follows:

Amount of Claims	**Probability**
25	0.25
50	0.50
100	0.15
500	0.05
1000	0.05

Calculate $f_S(100)$, the probability that aggregate claims under a policy will be 100 during a one year period.

Solution:

$f_S(100)$ will occur if we have:

Number of Claims	Amounts of Claims	Probability
1	100	$(0.25)(0.15)=0.0375$
2	50,50	$(0.60)(0.50)^2=0.15$
3	25,25,50	$(0.08)(0.25)^2(0.50)(3)=0.0075$
4	All 25	$(0.02)(0.25)^4=0.000078125$
Total➔		**0.195078125**