STAT 479
Test 2
Fall 2022
March 24, 2022

1. For Dental Insurance with no deductible, number of losses (N^L) is distributed as a Negative Binomial with $\gamma = 2$ and $\beta = 1$. The amount of the loss is distributed as an exponential distribution with $\theta = 200$.

   Deuschle Dental offers identical Dental Insurance except for an ordinary deductible of D.

   Let N^P be the random variable representing the number of payments under the Dental Insurance offered by Deuschle. The $Var[N^P] = 2.625$.

   Calculate D .

   Solution:

   $N^P \sim$ Negative Binommial with $\gamma = 2$ and $\beta = v(1)$

   $v = 1 - F(D)$

   $Var[N^P] = \gamma(\beta)(1+\beta) = (2)(v(1))(1+v(1)) = 2.625$

   $2v + 2v^2 = 2.625$

   $$2v^2 + 2v - 2.625 = 0 \Longrightarrow v = \frac{-2 \pm \sqrt{(2)^2 - (4)(2)(-2.625)}}{2(2)} = 0.75$$

   $$1 - F(D) = 0.75 \Longrightarrow e^{-D/200} = 0.75 \Longrightarrow D = 200(-\ln(0.75)) = 57.54$$

2. In an experiment, there are three die in bowl. There are two four sided die with numbers 1, 2, 3, and 4 on the faces of each four sided die. There is also one six sided die with numbers 1, 2, 3, 4, 5, and 6 on the faces of the six sided die.

   Let X be the number rolled for a die. You are given:

   - For the 4 sided die -- $E[X]=2.5$ and $Var[X]=\frac{5}{4}$
   - For the 6 sided die -- $E[X]=3.5$ and $Var[X]=\frac{35}{12}$

   Jackson randomly selects a die and rolls it. He records the value on the face of the die.

   a. Calculate the Expected Process Variance.

      Solution:

      $$EPV=\left(\frac{2}{3}\right)\left(\frac{5}{4}\right)+\left(\frac{1}{3}\right)\left(\frac{35}{12}\right)=1.80556$$

   b. Calculate the Variance of the Hypothetical Mean.

      Solution:

      $$E(X)=\left(\frac{2}{3}\right)(2.5)+\left(\frac{1}{3}\right)(3.5)=2.83333$$

      $$VHM=\left(\frac{2}{3}\right)(2.5-2.83333)^2+\left(\frac{1}{3}\right)(3.5-2.83333)^2=0.22222$$

   c. If you are using Buhlmann Credibility, calculate Z based on this one roll.

      Solution:

      $$Z=\frac{N}{N+K}$$

      $$K=\frac{EPV}{VHM}=\frac{1.08556}{0.22222}=8.125$$

      $$N=\frac{1}{1+8.125}=0.1096$$

3. The frequency of claims is assumed to follow a Poisson distribution.

   a. Under Classical Credibility, calculate the Full Credibility Criterion for Frequency such that the chance of being within 3% of the true mean in 88.12%.

      Solution:

$$\Phi(y) = \frac{1+P}{2} = \frac{1+0.8812}{2} = 0.9406 \text{ ==> } y = 1.56$$

$$n_0 = \left(\frac{1.56}{0.03}\right)^2 = 2704$$

The severity of the claims is assumed to be Gamma with $\alpha = 4$ and $\theta = 1000$.

   b. Under Classical Credibility, calculate the Full Credibility Criterion for Severity such that the chance of being within 3% of the true mean in 88.12%.

      Solution:

$$N = n_0\left(\frac{\sigma}{\mu}\right)^2 = (2704)\left(\frac{(1000)^2(4)}{[(4)(1000)]^2}\right) = 676$$

4. The following sample is assumed to be drawn from a uniform distribution over the range of 0 to U :

1000 2000 545

Calculate the Maximum Likelihood Estimator for U .

Solution:

$\hat{U} = Max\{1000, 2000, 525\} = 2000$

5. The claims for a hospital indemnity policy with an upper limit of 1000 is assumed to be from an exponential distribution with a parameter of θ. You have the following sample of claim payments:

100 200 400 1000 1000

Calculate the Maximum Likelihood Estimator for θ .

Solution:

$$\hat{\theta} = \frac{\text{Sum of all values}}{\text{Number of uncensored values}} = \frac{100 + 200 + 400 + 1000 + 1000}{3} = 900$$

6. Ding Dental Company sells a dental insurance policy with an upper limit per claim of 1000. There is no deductible and no coinsurance.

The first two claims received resulted in payments of:

100 and 1000

Ding believes that the total claim amount is distributed as a Weibull distribution with parameters $\tau = 2$ and θ.

Ding uses the maximum likelihood estimate to estimate θ.

a. Show that $L(\theta) = \dfrac{200e^{-\frac{1,010,000}{\theta^2}}}{\theta^2}$.

Solution:

$$L(\theta) = f(100)(1 - F(1000)) = \frac{2\left(\frac{100}{\theta}\right)^2 e^{-\left(\frac{100}{\theta}\right)^2}}{100} \cdot e^{-\left(\frac{1000}{\theta}\right)^2}$$

$$2\left(\frac{200}{\theta^2}\right) e^{-\left(\frac{100}{\theta}\right)^2 - \left(\frac{1000}{\theta}\right)^2} =$$

$$\frac{200e^{-\frac{1,010,000}{\theta^2}}}{\theta^2}$$

b. Calculate the Maximum Likelihood Estimate of θ.

Solution:

$$l(\theta) = \ln(200) - 1,010,000\theta^{-2} - 2\ln(\theta)$$

$$l'(\theta) = 0 - (-2)(1,010,000)\theta^{-3} - 2/\theta = 0$$

$$2,020,000 - 2\theta^2 = 0 ==> \hat{\theta} = \sqrt{\frac{2,020,000}{2}} = 1004.99$$

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7. The number of claims under an automobile policy is assumed to be distributed as a Poisson with a parameter of λ . You have the following information about a sample of claims from 10,000 drivers:

Number of Claims	Number of Drivers
0	5000
1	3000
2	1800
3	200

Calculate the 90% Linear Confidence Interval of the Maximum Likelihood Estimator of λ .

Solution:

$$\hat{\lambda} = \bar{X} = \frac{(0)(5000)+(1)(3000)+(2)(1800)+(3)(200)}{10,000} = 0.72$$

$$Var(\hat{\lambda}) = \frac{0.72}{10,000} = 0.000072$$

$$CI = 0.72 \pm (1.0645)(0.000072)^{0.5} = (0.70604; 0.73396)$$

8. Brett wants to test the following hypothesis using the Chi Square Test with a 97.5% significance level:

H_0: The data is from a Poisson distribution.
H_1: The data is not from a Poisson distribution.

Brett uses the data in the following table to complete the Chi Square Test:

Number of Accidents in 2021	Number of Policies
0	1600
1	3000
2	2600
3+	2800

Using this data, the Maximum Likelihood Estimator of λ is 1.8 .

(a) Calculate the Chi Square test statistic.

Solution:

j	**Observed**	**Expected**	$\chi^2 = \frac{(O_j - E_j)^2}{E_j}$
0	**1600**	$(10,000)p_0 = (10,000)\left(\frac{e^{-1.8}(1.8)^0}{0!}\right) = 1652.99$	**1.6986**
1	**3000**	$(10,000)p_0 = (10,000)\left(\frac{e^{-1.8}(1.8)^1}{1!}\right) = 2975.38$	**0.2037**
2	**2600**	$(10,000)p_0 = (10,000)\left(\frac{e^{-1.8}(1.8)^2}{2!}\right) = 2677.65$	**2.2628**
3	**2800**	$10,000 - 1652.99 - 2975.38 - 2677.64$ $= 2693.79$	**4.1877**
Total			**8.3528**

(b) Calculate the critical value for this test.
Solution:

$$df = 4 - 1 - 1 = 2 \Longrightarrow Critical\ Value = 7.378$$

(c) State Brett's conclusion.
Solution:

We reject H_0 since $\chi^2 > 7.378$

9. You are given the following claim amounts:

10 21 35 45 55

Your hypotheses are:

H_0: The data is from an exponential distribution with a mean of 40.

H_1: The data is not from an exponential distribution with a mean of 40.

Your boss completed the following work before heading to a meeting. He asked you to complete the work and answer the following questions.

a. Complete this table. (Show your work.)

x	$F_5(x^-)$	$F_5(x)$	$F^*(x)$	K-S Value
10	0	0.2	0.221	0.221
21	0.2	0.4	0.408	0.208
35	0.4	0.6	0.583	0.183
45	0.6	0.8	$1-e^{-\left[\frac{45}{40}\right]}=0.67535$	$0.8-0.67535$ $=0.12465$
55	0.8	1	$1-e^{-\left[\frac{55}{40}\right]}=0.74716$	$1-0.74716$ $=0.25284$

b. Calculate the Kolmogorov-Smirnov test Statistic to test this hypothesis.

Solution:

$$Max\{K-S\ Value\}=0.25284$$

c. State the critical value at a 95% confidence level.

Solution:

$$CriticalValue=\frac{1.36}{\sqrt{5}}=0.6082$$

d. State your conclusion from this Hypothesis Test.

Solution:

We fail to reject H_0 since $KS<0.6082$.

10. You are given the following sample of 10 claim amounts:

110, 120, 120, 125, 130, 150, 165, 175, 190, 300

Determine $F(125)$ for a smoothed empirical distribution.

Solution:

$$F(125) = \frac{i}{n+1} = \frac{4}{10+1} = \frac{4}{11}$$

11. The random variable X is distributed as a Pareto distribution with $\alpha = 5$ and $\theta = 800$.

You want to discretize this distribution using a span of 200.

a. Calculate the probability associated with the discrete value of 400 using the method of mass dispersal.

Solution:

$$Answer = F(500) - F(300)$$

$$= \left(1 - \left(\frac{800}{800+500}\right)^5\right) - \left(1 - \left(\frac{800}{800+300}\right)^5\right) = 0.1152$$

b. Calculate the probability associated with the discrete value of 400 using the method of moment matching where you match the mean.

Solution:

$$Answer = \frac{2E[X \wedge 400] - E[X \wedge 200] - E[X \wedge 600]}{200}$$

$$= \frac{2\left(\frac{800}{4}\left(1 - \left(\frac{800}{800+400}\right)^4\right)\right) - \left(\frac{800}{4}\left(1 - \left(\frac{800}{800+200}\right)^4\right)\right) - \left(\frac{800}{4}\left(1 - \left(\frac{800}{800+600}\right)^4\right)\right)}{200}$$

$$= 0.12116$$

12. The following sample of four claims were received during 2021:

1000 5000 14,000 100,000

These losses are assumed to be distributed as a normal distribution with parameters of μ and σ. Determine the maximum likelihood estimates of μ and σ.

Solution:

$$\hat{\mu} = \bar{X} = \frac{1000 + 500 + 14,000 + 100,000}{4} = 30,000$$

$$\hat{\sigma} = \sqrt{Var(X)}$$

$$= \sqrt{\frac{(1000 - 30,000)^2 + (5000 - 30,000)^2 + (14,000 - 30,000)^2 + (100,000 - 30,000)^2}{4}}$$

$$= 40,687.84$$