

**STAT 479**

**Test 2**

**Fall 2022**

March 24, 2022

1. For Dental Insurance with no deductible, number of losses ( $N^L$ ) is distributed as a Negative Binomial with  $\gamma = 2$  and  $\beta = 1$ . The amount of the loss is distributed as an exponential distribution with  $\theta = 200$ .

Deuschle Dental offers identical Dental Insurance except for an ordinary deductible of  $D$ .

Let  $N^P$  be the random variable representing the number of payments under the Dental Insurance offered by Deuschle. The  $\text{Var}[N^P] = 2.625$ .

Calculate  $D$ .

2. In an experiment, there are three die in bowl. There are two four sided die with numbers 1, 2, 3, and 4 on the faces of each four sided die. There is also one six sided die with numbers 1, 2, 3, 4, 5, and 6 on the faces of the six sided die.

Let  $X$  be the number rolled for a die. You are given:

- For the 4 sided die --  $E[X] = 2.5$  and  $Var[X] = \frac{5}{4}$
- For the 6 sided die --  $E[X] = 3.5$  and  $Var[X] = \frac{35}{12}$

Jackson randomly selects a die and rolls it. He records the value on the face of the die.

- a. Calculate the Expected Process Variance.

- b. Calculate the Variance of the Hypothetical Mean.

- c. If you are using Buhlmann Credibility, calculate  $Z$  based on this one roll.

3. The frequency of claims is assumed to follow a Poisson distribution.
  - a. Under Classical Credibility, calculate the Full Credibility Criterion for Frequency such that the chance of being within 3% of the true mean is 88.12%.

The severity of the claims is assumed to be Gamma with  $\alpha = 4$  and  $\theta = 1000$ .

- b. Under Classical Credibility, calculate the Full Credibility Criterion for Severity such that the chance of being within 3% of the true mean is 88.12%.

4. The following sample is assumed to be drawn from a uniform distribution over the range of 0 to  $U$  :

1000 2000 545

Calculate the Maximum Likelihood Estimator for  $U$  .

5. The claims for a hospital indemnity policy with an upper limit of 1000 is assumed to be from an exponential distribution with a parameter of  $\theta$ . You have the following sample of claim payments:

100 200 400 1000 1000

Calculate the Maximum Likelihood Estimator for  $\theta$ .

6. Ding Dental Company sells a dental insurance policy with an upper limit per claim of 1000. There is no deductible and no coinsurance.

The first two claims received resulted in payments of:

100 and 1000

Ding believes that the total claim amount is distributed as a Weibull distribution with parameters  $\tau = 2$  and  $\theta$ .

Ding uses the maximum likelihood estimate to estimate  $\theta$ .

a. Show that  $L(\theta) = \frac{200e^{-\frac{1,010,000}{\theta^2}}}{\theta^2}$ .

- b. Calculate the Maximum Likelihood Estimate of  $\theta$ .

7. The number of claims under an automobile policy is assumed to be distributed as a Poisson with a parameter of  $\lambda$ . You have the following information about a sample of claims from 10,000 drivers:

Number of Claims	Number of Drivers
0	5000
1	3000
2	1800
3	200

Calculate the 90% Linear Confidence Interval of the Maximum Likelihood Estimator of  $\lambda$ .

8. Brett wants to test the following hypothesis using the Chi Square Test with a 97.5% significance level:

$H_0$ : The data is from a Poisson distribution.

$H_1$ : The data is not from a Poisson distribution.

Brett uses the data in the following table to complete the Chi Square Test:

Number of Accidents in 2021	Number of Policies
0	1600
1	3000
2	2600
3+	2800

Using this data, the Maximum Likelihood Estimator of  $\lambda$  is 1.8 .

- (a) Calculate the Chi Square test statistic.

- (b) Calculate the critical value for this test.

- (c) State Brett's conclusion.



9. You are given the following claim amounts:

10 21 35 45 55

Your hypotheses are:

$H_0$ : The data is from an exponential distribution with a mean of 40.

$H_1$ : The data is not from an exponential distribution with a mean of 40.

Your boss completed the following work before heading to a meeting. He asked you to complete the work and answer the following questions.

a. Complete this table. (Show your work.)

$x$	$F_5(x^-)$	$F_5(x)$	$F^*(x)$	K-S Value
10	0	0.2	0.221	0.221
21	0.2	0.4	0.408	0.208
35	0.4	0.6	0.583	0.183
45				
55				

b. Calculate the Kolmogorov-Smirnov test Statistic to test this hypothesis.

c. State the critical value at a 95% confidence level.

d. State your conclusion from this Hypothesis Test.

10. You are given the following sample of 10 claim amounts:

110, 120, 120, 125, 130, 150, 165, 175, 190, 300

Determine  $F(125)$  for a smoothed empirical distribution.

11. The random variable  $X$  is distributed as a Pareto distribution with  $\alpha = 5$  and  $\theta = 800$ .

You want to discretize this distribution using a span of 200.

- a. Calculate the probability associated with the discrete value of 400 using the method of mass dispersal.

- b. Calculate the probability associated with the discrete value of 400 using the method of moment matching where you match the mean.

12. The following sample of four claims were received during 2021:

1000 5000 14,000 100,000

These losses are assumed to be distributed as a normal distribution with parameters of  $\mu$  and  $\sigma$ . Determine the maximum likelihood estimates of  $\mu$  and  $\sigma$ .