STAT 479
Test 3
Spring 2022

May 3, 2022

1. Barwegen Auto Company sells automobile insurance. Barwegen splits drivers into three categories – Safe, Not So Safe, and Crazy. You have the following information:

Type	Proportion of Total Drivers	Claim Frequency Poisson Annual	Severity Gamma
Safe	0.5	0.10	$\alpha = 2, \theta = 500$
Not So Safe	0.4	0.25	$\alpha = 4, \theta = 500$
Crazy	0.1	0.50	$\alpha = 6, \theta = 500$

For each driver, frequency and severity are independent.

During 2021, five drivers are selected randomly from one type. During 2021, these five drivers have one claim for an amount of 2500.

a. (5 points) What is the expected value of the process variance of the **claim severity**?

Solution:

	Expected Accidents	Variance of X	EPV
Safe	(0.5)(0.1) = 0.05	$(2)(500)^2$ = 500,000	(0.05/0.20)(500,000) = 125,000
No So Safe	(0.4)(0.25) = 0.10	$(4)(500)^2$ = 1,000,000	(0.10/0.20)(1,000,000) = 500,000
Crazy	(0.1)(0.50) = 0.05	$(3)(500)^2$ = 1,500,000	(0.05/0.20)(1,500,000) = 375,000
Total	0.20		**1,000,000**

Answer = 1,000,000

b. (5 points) What is the variance of the hypothetical mean of the **claim severity**?

Solution:

$$E[X] = (0.05/0.20)(2)(500) + (0.10/0.20)(4)(500) + (0.05/0.20)(6)(500) = 2000$$

$$E[X^2] = (0.05/0.20)(1000)^2 + (0.10/0.20)(2000)^2 + (0.05/0.20)(3000)^2 = 4,500,000$$

$$VHM = 4,500,000 - (2000)^2 = 500,000$$

c. (5 points) Use Buhlmann Credibility to estimate the amount of claims in 2022 per claim for these five drivers.

Solution:

$$K = \frac{EPV}{VHM} = \frac{1,000,000}{500,000} = 2 \qquad Z = \frac{N}{N+K} = \frac{1}{1+2} = 1/3$$

$$\mu = 2000 \qquad \bar{X} = 2500$$

$$\text{Estimated Severity} = \left(\frac{1}{3}\right)(2500) + \left(\frac{2}{3}\right)(2000) = 2166.67$$

2. (10 points) Three risks are selected at random from a population and observed for 2 years. The risks had the following number of claims over those two years:

Risk	Number of Claims	
	Year 1	**Year 2**
Risk 1	0	2
Risk 2	2	2
Risk 3	0	0

Use the nonparametric empirical Bayes procedure to calculate the credibility weighted estimates of expected number of claims for Risk 2 in year 3.

Solution:

$$\bar{X}_1 = \frac{2}{2} = 1 \qquad \bar{X}_2 = \frac{4}{2} = 2 \qquad \bar{X}_3 = \frac{0}{2} = 0$$

$$\bar{X} = \mu = \frac{0+2+2+2+0+0}{6} = 1 \quad \text{or} \quad \bar{X} = \mu = \frac{1+2+0}{3} = 1$$

$$\hat{\sigma}_1^2 = \frac{(0-1)^2 + (2-1)^2}{2-1} = 2 \qquad \hat{\sigma}_2^2 = \frac{(2-2)^2 + (2-2)^2}{2-1} = 0$$

$$\hat{\sigma}_3^2 = \frac{(0-0)^2 + (0-0)^2}{2-1} = 0 \qquad EPV = \frac{2+0+0}{3} = \frac{2}{3}$$

$$Var(X) = \frac{(1-1)^2 + (2-1)^2 + (0-1)^2}{3-1} = 1$$

$$\widehat{VHM} = \widehat{Var}(X) - \frac{\widehat{EPV}}{3-1} = 1 - \frac{2/3}{2} = \frac{2}{3}$$

$$\hat{K} = \frac{\widehat{EPV}}{\widehat{VHM}} = \frac{2/3}{2/3} = 1 \qquad Z = \frac{N}{N+K} = \frac{2}{2+1} = 2/3$$

Expected number of claims for Risk 2 in year 3

$$= \left(\frac{2}{3}\right)(\bar{X}_2) + \left(\frac{1}{3}\right)(\mu) = \left(\frac{2}{3}\right)(2) + \left(\frac{1}{3}\right)(1) = \frac{5}{3}$$

3. (10 points) There are three types of risks. Each risk has a different chance of a claim. Each risk has either zero claims or one claim. The information regarding risks are summarized in the table below:

Type of Risk	A Priori Chance of Type of Risk	Chance of Claim
1	50%	10%
2	30%	30%
3	20%	50%

A risk is selected at random. You observe zero claims in a year. Use Bayesian Analysis to determine the expected annual claim frequency for the same risk during the next year?

Solution:

Type	Prior	P(Zero Claims)	Weighted Probability	Posterior
1	0.5	0.9	(0.5)(0.9) = 0.45	0.45/0.76
2	0.3	0.7	(0.3)(0.7) = 0.21	0.21/0.76
3	0.2	0.5	(0.2)(0.5) = 0.10	0.10/0.76
Total			0.76	

$$\text{Annual Claim Frequency} = \left(\frac{0.45}{0.76}\right)(0.1) + \left(\frac{0.21}{0.76}\right)(0.3) + \left(\frac{0.10}{0.76}\right)(0.5) = 0.2079$$

4. The number of claims that a particular insured makes in a year follows a Poisson distribution with a mean of λ. The value of λ for the population of insureds follows a Gamma distribution with $\alpha = 4$ and $\theta = \frac{1}{20}$. Maren is chosen at random from the policyholders.

a. (5 points) Calculate the probability that Maren will have one claims in the next year.

Solution:

Claims are distributed Negative Binomial with $\gamma = \alpha = 4$ and $\beta = \theta = \frac{1}{20}$

$$p_1 = \frac{\gamma\beta}{(1+\beta)^{\gamma+1}} = \frac{(4)(1/20)}{(1+1/20)^5} = 0.1567$$

b. (5 points) Maren is observed for 3 years. She has 3 claims in the first year, zero claims in the second year and 2 claims in the third year. Calculate the expected number of claims for the next year for Maren using the posterior distribution.

Solution:

$Y = 3 \qquad C = 5$

$$\alpha' = \alpha + C = 4 + 5 = 9 \qquad \theta' = \frac{\theta}{Y\theta + 1} = \frac{1/20}{3/20 + 1} = 1/23$$

Claims are distributed Negative Binomial with $\gamma = \alpha' = 9$ and $\beta = \theta' = \frac{1}{23}$

$$E[N] = \gamma\beta = (9)(1/23) = \frac{9}{23}$$

5. (10 points) You are given the following Paid Claims triangle:

Incremental Loss Payments by Year				
Year of Service	Year of Payment			
	Year 0	Year 1	Year 2	Year 3
2018	25,000	15,000	10,000	5,000
2019	30,000	20,000	14,000	
2020	40,000	30,000		
2021	50,000			

There is no claims paid after Year 3.

Calculate the loss reserve on December 31, 2021 using the chain ladder method with **volume weighted** average loss development factors.

Solutions:

Cumulative Loss Payments by Year				
Year of Service	Year of Payment			
	Year 0	Year 1	Year 2	Year 3
2018	25,000	40,000	50,000	55,000
2019	30,000	50,000	64,000	
2020	40,000	70,000		
2021	50,000			

$$\text{Loss Development Factors} \Longrightarrow\ 1/0 = \frac{40,000+50,000+70,000}{25,000+30,000+40,000} = 1.68421$$

$$2/1 = \frac{50,000+64,000}{40,000+50,000} = 1.26667$$

$$3/2 = \frac{55,000}{50,000} = 1.1$$

Reserves

2018: No Reserves

$2019:\ 64,000(1.1) - 64,000 = 6400$

$2020:\ 70,000(1.026667)(1.1) - 70,000 = 27,533.33$

$2021:\ 50,000(1.68421)(1.226667)(1.1) - 50,000 = 67,333.33$

$$Total = 6400 + 27,533.33 + 67,333.33 = 101,266.67$$

6. During 2021, Moody Stop Loss Company sold 4 policies. Each policy had an annual premium. The date of the policy and the amount of the annual premium is listed in the following table:

Policy	Date of Policy	Annual Premium
Policy 1	01/15/2021	120,000
Policy 2	04/01/2021	30,000
Policy 3	05/01/2021	180,000
Policy 4	10/1/2021	40,000

a. (5 points) Determine the total unearned premium for these policies on December 31, 2021.

Solution:

$$Unearned = \left(\frac{0.5}{12}\right)(120,000) + \left(\frac{3}{12}\right)(30,000) + \left(\frac{4}{12}\right)(180,000) + \left(\frac{9}{12}\right)(40,000)$$

$$= 102,500$$

b. (5 points) During 2021, Moody has paid claims of 13,000. Moody expects the loss ratio on these policies to be 72%. Use the loss ratio method to determine the loss reserve at December 31, 2021.

Solution:

$$\text{Earned Premium} = 370,000 - 102,500 = 267,500$$

$$\text{Expected ultimate losses} = (267,500)(0.72) = 192,600$$

$$\text{Reserves} = 192,600 - 13,000 = 179,600$$

7. (10 points) For medical insurance policies issued in 2019, the earned premium is 1,000,000. The claims paid in 2019, 2020, and 2021 total 550,000. The expected loss ratio is 0.70. The loss reserve on December 31, 2021 using the Bornhuetter-Ferguson method is 135,484.

   Determine the loss reserve as of December 31, 2021 using the Chain Ladder method.

   Solution:

$$\text{Expected Ultimate Losses} = (1{,}000{,}000)(0.70) = 700{,}000$$

$$\text{B. F. Method} = 135{,}484 = 700{,}000\left(1 - \frac{1}{f_{ULT}}\right)$$

$$f_{ULT} = 1.24$$

$$\text{Chain Ladder} = (550{,}000)(1.24) - 550{,}000 = 132{,}000$$

8. (10 points) You are given the following projected loss reserve development.

Estimated Paid Losses and Loss Reserves by Accident Year Based on Average Paid Loss Developmment Factors Derived Above						
	Development Year					
Accident Year	1	2	3	Estimated Ultimate Losses	Paid to Date	Estimated Loss Reserve
2016				31,000	31,000	0
2017			47,034	47,034	44,000	3,034
2018		42,881	45,838	45,838	37,000	8,838
2019	50,467	58,488	62,522	62,522	30,000	32,522
Total				186,395	142,000	44,395

Using an annual effective interest rate of 8% and assuming future loss payments are made in the middle of the year, calculate the discounted reserve using the values above.

Solution:

	Incremental Reserves		
	1	2	3
2017			47,034 - 44,000 = 3034
2018		42,881 – 37,000 = 3881	45,838 – 42,881 = 2957
2019	50,467 – 30,000 = 10,467	58,488 – 50,467 = 8,021	62,522 – 58,488 = 4034

$$\text{Discounted Reserves} = (3034+5881+20,467)(1.08)^{-0.5} + (8021+2957)(1.08)^{-1.5} + (4034)(1.08)^{-2.5}$$

$$= 41,382$$

9. (10 points) Luke is the rate making actuary for Andrew Auto. He is setting rates for the auto coverage which is a short term insurance product. You are given the following data:

Calendar Year	Earned Premium
2019	30,000
2020	40,000
2021	50,000

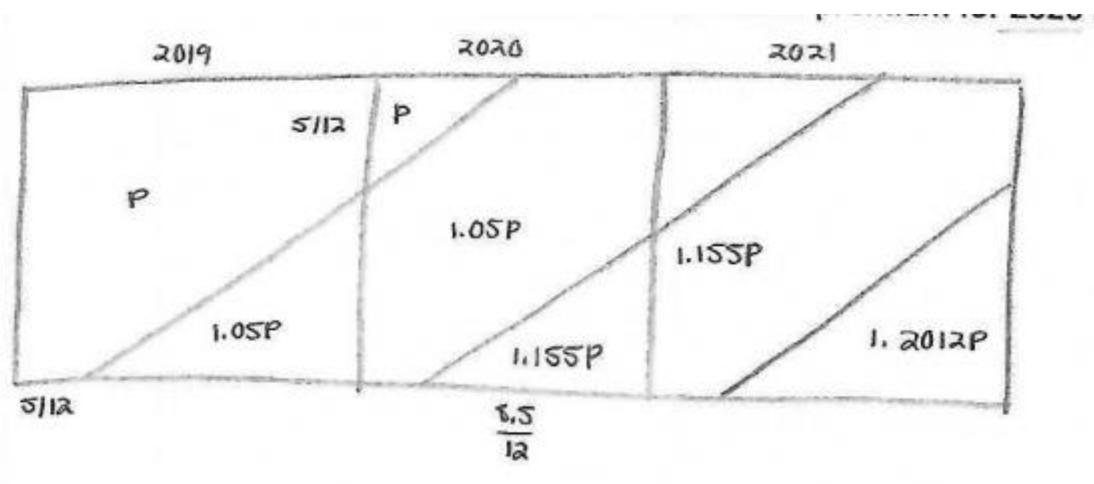
Assume that all policies are one year policies and the policies are issued uniformly throughout the year.

The following rate changes have occurred:

Date	Rate Change
June 1, 2019	5% Increase
April 15, 2020	10% Increase
February 1, 2021	4% Increase

Using the parallelogram method, calculate the earned premium for 2020 based on current rates.

Solutions:



$2020:$

$$\left(\frac{1}{2}\right)\left(\frac{5}{12}\right)^2 = 0.0868 @ P$$

$$\left(\frac{1}{2}\right)\left(\frac{8.5}{12}\right)^2 = 0.2509 @ (1.05)(1.10)P = 1.155P$$

$$[1 - 0.0868 - 0.2509] = 0.6623 @ 1.05P$$

Weighted Earned Premium = $0.0868P + 0.2509(1.155P) + 0.6623(1.05P) = 1.072P$

Current Rates = (1.05)(1.10)(1.04) = 1.2012P

Earned Premium at current rates = $(40,000)\left(\frac{1.2012}{1.072}\right) = 44,820.86$

10. Alec is the ratemaking actuary for Maple Medical Insurance. Maple has the following data to use to determine the new average gross premium rate for 2022:

Expected Effective Period Incurred Losses	25,000,000
Earned Exposure Units	20,000
Earned Premium at Current Rates	30,000,000
Premium Per Unit	1500
Fixed Expenses	3,000,000
Permissible Loss Ratio	0.85

a. (5 points) The new average gross premium rate is 1650 to the nearest 10. Calculate the new average gross premium rate to the nearest 1.

Solution:

$$\frac{\frac{25,000,000}{20,000}+\frac{3,000,000}{20,000}}{0.85}=1647$$

Maple sells medical insurance in Indiana, Illinois, and Ohio. During 2021, Maple used the following differentials and had the following experience:

State	2021 Differential	Loss Ratio in 2021	Exposure Units
Indiana	0.80	0.72	5,000
Illinois	1.00	0.8	10,000
Ohio	1.20	0.76	5,000

b. (5 points) Determine the new differentials for 2022.

Solution:

$$\text{Indiana: (2021 Differential)}\left(\frac{\text{Indiana LR}}{\text{Illinois LR}}\right)=(0.80)\left(\frac{0.72}{0.80}\right)=0.72$$

Illinois: 1.00 since this is the base case

$$\text{Ohio: (2021 Differential)}\left(\frac{\text{Ohio LR}}{\text{Illinois LR}}\right)=(1.20)\left(\frac{0.76}{0.80}\right)=1.14$$

c. (5 points) Determine the revised increase in the base premium rate for 2022 so that the gross premium change and the differentials are balanced.

Solution:

$$\text{Off Balance Factor} = \frac{(0.72)(5000)+(1.00)(10,000)+(1.14)(5000)}{(0.80)(5000)+(1.00)(10,000)+(1.20)(5000)} = 0.965$$

$$\text{Goal Increase} = \frac{1647}{1500} = 1.098$$

$$\text{Revised Increase} = \frac{1.098}{0.965} = 1.1378$$

11. (10 points) Purdue Auto Insurance Company sells liability insurance with limits of 250,000, 1,000,000, and 5,000,000. The base limit is 250,000.

The Company has the following experience for the last three years:

Size of Loss	**Number of Claims**	**Ground Up Total Losses (in Millions)**
1 – 250,000	1200	180.00
250,001 – 1,000,000	850	361.25
1,000,001 – 5,000,000	150	187.50
Total	2200	728.75

Calculate the increased limit factors (ILFs) for the limits of 1,000,000 and 5,000,000.

Solution:

Size of Loss	**250,000 Limit**	**1,000,000 Limit**	**5,000,000 Limit**
1-250,000	180.00	180.00	180.00
250,001 – 1,000,000	(850)(0.250) = 212.50	361.25	361.25
1,000,001 – 5,000,000	(150)(0.250) = 37.50	(150)(1) = 150	187.50
Total	430	691.25	728.75

$$ILF_{1,000,000} = \frac{691.25}{430} = 1.6076$$

$$ILF_{5,000,000} = \frac{728.75}{430} = 1.6948$$

12. (10 points) There are six ingredients to rate making. Please list them below.

    Solution:

    Incurred Losses

    Trend

    Credibility

    Expenses

    Loading for contingencies and profit

    Investment income

13. In the rate making process, it is important to "trend" our claims.

a. (3 points) What is trend?

Solution:

Trend is the increase in claim cost generally do to inflation.

The Loss Cost Per Exposure Unit is 900 for the period from July 1, 2017 to December 31, 2018. You want to estimate the Loss Cost Per Exposure Unit for rates to be effective from August 1, 2019 to December 31, 2020. You expect the exponential trend rate will be 6%.

b. (7 points) Determine Loss Cost Per Exposure Unit based on using the midpoint rule.

Solution:

The midpoint of the exposure period was April 1, 2018.

The midpoint of the rate period is April 15, 2020.

This is 24.5 months

$$Answer = 900e^{0.06\left(\frac{24.5}{12}\right)} = 1017.29$$

An alternative solution would be as follows since the question is not clear. This answer assumes all rates are annual rates and that there are uniform distribution of issues.

The midpoint of the exposure period was April 1, 2018.

The midpoint of the rate period is October 15, 2020.

This is 30.5 months

$$Answer = 900e^{0.06\left(\frac{30.5}{12}\right)} = 1048.27$$

14. (10 points) The following developed losses are given:

   i. The total ground up losses per policy with a 100,000 limit is 500.

   ii. The total ground up losses per policy with a 250,000 limit is 700.

The base rate at the 100,000 limit is 600 which consists of 540 of pure premium, 40 of fixed expenses, and 20 of variable expenses.

Calculate the premium rate for the 250,000 limit.

Solution:

$$ILF_{250,000} = \frac{700}{500} = 1.4$$

$$\text{Pure Premium} = (540)(1.4) = 756$$

$$\text{Premium} = \frac{756 + 40}{1 - \frac{20}{600}} = 823.45$$