STAT 490
Spring 2015
Quiz 2
March 5, 2015

1. A two-year temporary life annuity due provides a payment of 1000 at time 0 and another payment of 1000 at time 1 if (x) is alive.

   You are given:

   i. $v = 0.95$

   ii. Mortality is a random variable with the following distribution:

Mortality Rate for (x)	Probability
0.08	0.30
0.10	0.60
0.12	0.10

   iii. Y is the present value random variable for this annuity.

   a. (7 points) Calculate the $E[Y]$.

   Solution:

$$PV(q_x = 0.08) = 1000 + 1000(0.95)(1 - 0.08) = 1874$$

$$PV(q_x = 0.10) = 1000 + 1000(0.95)(1 - 0.10) = 1855$$

$$PV(q_x = 0.12) = 1000 + 1000(0.95)(1 - 0.12) = 1836$$

$$EPV = (0.3)(1874) + (0.6)(1855) + (0.1)(1836) = 1858.80$$

b. (13 points) Calculate the $Var[Y]$.

Solution:

$$PV(q_x = 0.08 \,\&\, Die) = 1000 \implies \Pr = (0.3)(0.08) = 0.024$$

$$PV(q_x = 0.08 \,\&\, Live) = 1000 + 1000(0.95) = 1950 \implies \Pr = (0.3)(0.92) = 0.276$$

$$PV(q_x = 0.10 \,\&\, Die) = 1000 \implies \Pr = (0.6)(0.1) = 0.06$$

$$PV(q_x = 0.10 \,\&\, Live) = 1000 + 1000(0.95) = 1950 \implies \Pr = (0.6)(0.90) = 0.54$$

$$PV(q_x = 0.12 \,\&\, Die) = 1000 \implies \Pr = (0.1)(0.12) = 0.012$$

$$PV(q_x = 0.12 \,\&\, Live) = 1000 + 1000(0.95) = 1950 \implies \Pr = (0.1)(0.88) = 0.088$$

Now regroup

PV=1000 with Pr = 0.096 and PV=1950 with Pr = 0.904

$$Var[Y] = E[Y^2] - \left(E[Y]\right)^2 = (1000)^2(0.096) + (1950)^2(0.904) - (1858.8)^2 = 78,322.56$$