1. A fully discrete whole life on (100) pays a death benefit of 80,000. Premiums are paid annually.

You are given:

i. Mortality is as follows:

x	q_x	l_x	d_x
100	0.25	1000	250
101	0.50	750	375
102	0.80	375	300
103	1.00	75	0

ii. The following spot interest rate curve:

t	y_t
1	0.0400
2	0.0500
3	0.0575
4	0.0625
5	0.0650

a. (7 points) Calculate the net benefit premium for this policy.

Solution:

$$P\frac{80,000\left[\frac{250}{1.04}+\frac{375}{(1.05)^2}+\frac{300}{(1.0575)^3}+\frac{75}{(1.0625)^4}\right]}{1000+\frac{750}{1.04}+\frac{375}{(1.05)^2}+\frac{75}{(1.0575)^3}}=33,625.20$$

b. (8 points) Calculate the net benefit reserve at the end of the second year.

Solution:

$${}_1V=\frac{({}_0V+P)(1+f_{[0,1]})-80,000(q_{100})}{p_{100}}=\frac{(0+33,625.20)(1.04)-80,000(0.25)}{0.75}=19,960.27$$

$${}_2V=\frac{({}_1V+P)(1+f_{[1,2]})-80,000(q_{101})}{p_{101}}=$$

$$\frac{(19,960.27+33,625.20)\left(\frac{(1.05)^2}{1.04}\right)-80,000(0.5)}{0.5}=33,611.51$$

2. Purdue Life Insurance Company has 1000 insureds who are exact age 80.

You are given that $q_{80} = 0.08$ and D is the random variable representing the number of deaths during the next year.

a. (4 points) Calculate the $E[D]$.

Solution:

$$E[D] = N(q_{80}) = (1000)(0.08) = 80$$

b. (5 points) Calculate the $Var[D]$.

Solution:

$$Var[D] = N(q_{80})(p_{80}) = (1000)(0.08)(0.92) = 73.6$$

c. (6 points) Purdue provides a death benefit of 10,000 to each of these 1000 insureds. Assuming the normal distribution, calculate the probability that the total death benefit that Purdue will pay over the next year will be more than 900,000.

Solution:

$$\text{Total Expected Death Benefit} = (80)(10,000)$$

$$\text{Total Variance} = (10,000)^2(73.6)$$

$$\Pr\left(Z > \frac{900,000 - (80)(10,000)}{\sqrt{(10,000)^2(73.6)}} \right) = \Pr(Z > 1.17) = 1 - 0.8790 = 0.121$$

3. A one-year term life insurance policy on (x) provides a death benefit of 1000 at the end of the year of death.

You are given:

i. $q_x = 0.10$

ii. The interest rate is a random variable with the following distribution:

Interest Rate	Probability
0.05	0.40
0.10	0.60

iii. Z is the present value random variable for this policy.

a. (5 points) Calculate the $E[Z]$.

b. (10 points) Calculate the $Var[Z]$.

Solution:

Potential Events:

Insured Dies Year 1; Interest Rate = 0.05	$\frac{1000}{1.05}$	Probability = (0.1)(0.4) = 0.04
Insured Dies Year 1; Interest Rate = 0.10	$\frac{1000}{1.10}$	Probability = (0.1)(0.6) = 0.06
Insured lives Year 1	0	Probability = 0.9

$$E[Z] = \left(\frac{1000}{1.05}\right)(0.04) + \left(\frac{1000}{1.10}\right)(0.06) = 92.64069$$

$$E[Z^2] = \left(\frac{1000}{1.05}\right)^2 (0.04) + \left(\frac{1000}{1.10}\right)^2 (0.06) = 85,867.956$$

$$Var[Z] = E[Z^2] - \left(E[Z]\right)^2 = 85,867.956 - (92.64069)^2 = 77,285.66$$

4. For a three year term insurance policy on (x), you are given the follow profits (profit vector) and premiums for each policy that is in force at the start of the policy year:

Time (t)	Profit	Premium at t-1	q_{x+t-1}	Withdrawal Rate at t
0	-50			
1	40	50	0.04	0.10
2	25	50	0.06	0.05
3	0	50	0.08	0.00

a. (5 points) Calculate the internal rate of return for this policy.

Solution:

Profit Signature = $-50; 40; 25(p_x^{(\tau)}) = 25(1-0.1)(1-0.4) = 21.6;$ and $0({}_2p_x^{(\tau)}) = 0$

$$-50 + 40(1+i)^{-1} + 21.6(1+i)^{-2} = 0$$

$$-50(1+i)^2 + 40(1+i)^1 + 21.6 = 0 \Longrightarrow 1+i = \frac{-40 \pm \sqrt{(-40)^2 - 4(-50)(21.6)}}{(2)(-50)} \Longrightarrow i = 16.94\%$$

b. (5 points) Calculate the Net Present Value of the profits using a 10% interest rate.

Solution:

$$NPV = -50 + 40(1.10)^{-1} + 21.6(1.10)^{-2} = 4.21488$$

c. (5 points) Calculate the profit margin for this policy using a 10% interest rate.

Solution:

$$PM = \frac{NPV}{\text{PV of Prem}} = \frac{4.21488}{50 + \frac{50(0.9)(0.96)}{1.10} + \frac{50(0.9)(0.96)(0.95)(0.94)}{(1.10)^2}} = \frac{4.21488}{121.15504} = 0.034789$$

5. (15 points) A fully discrete participating whole life on (50) provides a death benefit of 100,000. You are given:

   i. The gross premium paid annually is 2350.
   ii. The reserve at the end of the 9th year is 15,000.
   iii. The reserve at the end of the 10th year is 16,200.
   iv. The reserve at the end of the 11th year is 17,500.
   v. Cash values are 80% of the reserves
   vi. Mortality follows the Illustrative Life Table.
   vii. Withdrawals are 10% in the first year and 5% all year thereafter.
   viii. The interest rate earned is 8% in all years.
   ix. Issue expenses are 50% of premium.
   x. Maintenance expenses are 7% of premium plus 50 per policy in all years including the first year.
   xi. Dividends payable to all insureds alive at the beginning of the year are 70% of the profit for that year.

Calculate the dividend at the end of the 10th year.

Solution:

$${}_9V = 15,000; P_{10} = 2350$$

$$Exp_t = (2350)(0.1) + 50 = 214.50$$

$$DB_{10} = (100,000)(q_{50+9}) = (100,000)(0.01262) = 1262$$

$$WB_{10} = (16,200)(0.8)(1-0.01262)(0.05) = 639.82$$

$${}_{10}V \cdot p_{59}^{(\tau)} = (16,200)(1-0.01262)(1-0.05) = 15,195.78$$

$$\text{Profit} = (15,000 + 2350 - 214.5)(1.08) - 1262.00 - 639.82 - 15,195.78 = 1408.74$$

$$\text{Dividend} = (1408.74)(0.7) = 986.12$$

6. (15 points) You are given the following information for a Type B universal life contract:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Additional Death Benefit	Annual Cost of Insurance Charge	Annual Discount Rate for COI	Annual Credited Interest Rate
1	2000	40%	25	50,000	0.008	4%	5%
2	P_2	10%	25	50,000	0.012	4%	5.2%

The account value at the end of the second year is equal to the account value at the end of the first year.

Determine P_2.

Solution:

$$AV_1 = [AV_0 + P(1-f) - EC - COI](1+i^c) =$$

$$\left[0 + (2000)(1-0.4) - 25 - \left(\frac{50,000}{1.04}\right)(0.008)\right](1.05) = 829.90$$

$$AV_1 = [AV_1 + P_2(1-f) - EC - COI](1+i^c) =$$

$$\left[829.90 + (P_2)(1-0.1) - 25 - \left(\frac{50,000}{1.04}\right)(0.012)\right](1.052) = 829.90$$

$$==> 0.9P_2 = 560.9012138 ==> P_2 = 623.22$$