STAT 490
Spring 2016
Test 2
March 24, 2016

1. A life insurance company sells a three year term policy to (80) with a death benefit of 10,000.

   You are given that $q_{80} = 0.10$, $q_{81} = 0.2$ and $q_{82} = 0.3$.

   You are also given that the current spot interest rate curve:

Time t	y_t
1	0.03
2	0.04
3	0.05
4	0.06

   Premiums are paid annually.

   a. (10 points) The net benefit premium is 1800 to the nearest 100. Calculate the net benefit premium to the nearest 0.01.

      Solution:

$$\left(1000 + \frac{900}{1.03} + \frac{720}{1.04^2}\right)P = 10,000\left(\frac{100}{1.03} + \frac{180}{1.04^2} + \frac{216}{1.05^3}\right)$$

$$P = 1772.405152$$

   b. (10 points) Calculate the net benefit reserve at the end of two years.

      Solution:

$${}_0V = 0$$

$${}_1V = \frac{(0 + 1772.41)(1.03) - 10,000(0.1)}{1 - 0.1} = 917.313667$$

$${}_2V = \frac{(919.31 + 1772.41)\left(\frac{1.04^2}{1.03}\right) - 10,000(0.2)}{1 - 0.2} = 1030.5887$$

c. (15 points) Let L_0^n be the loss at issue random variable for this insurance. Calculate the $Var[L_0^n]$.

Solution:

$$\Pr(DieYr1) = 0.1 \rightarrow L_0^n = \frac{10,000}{1.03} - 1772.41 = 7936.328$$

$$\Pr(DieYr2) = 0.9(0.2) = 0.18 \rightarrow L_0^n = \frac{10,000}{1.04^2} - 1772.41\left(1 + \frac{1}{1.03}\right) = 5752.366$$

$$\Pr(DieYr3) = 0.9(0.8)(0.3) = 0.216 \rightarrow L_0^n = \frac{10,000}{1.05^3} - 1772.41\left(1 + \frac{1}{1.03} + \frac{1}{1.04^2}\right) = 3506.487$$

$$\Pr(LiveYr3) = 0.9(0.8)(0.7) = 0.504 \rightarrow L_0^n = -1772.41\left(1 + \frac{1}{1.03} + \frac{1}{1.04^2}\right) = -5131.889$$

$$E(L_0^n) = -0.012184$$

$$E\left[(L_0^n)^2\right] = 28,183,983.77$$

$$Var(L_0^n) = E\left[(L_0^n)^2\right] - \left[E(L_0^n)\right]^2 = 28,183,983.77$$

2. John Life Insurance Company sells a two year temporary life annuity due with annual payments of 100,000 to (x) .

You are given that $v = 0.9$.

You are also given that mortality is a random variable with the following distribution:

Value of q_x	Probability
0.08	0.4
0.13	0.6

Let Y be the present value random variable for this annuity.

a. (10 points) The $E[Y] = 180,000$ to the nearest 1000. Calculate the $E[Y]$ to the nearest 1.

Solution:

$$E(Y) = \Pr(DieYr1) * (100,000) + \Pr(LiveYr1) * (100,000) * (1 + 0.9)$$

$$= \left(0.4(.08) + 0.6(0.13)\right)\left(100,000\right) + \left(0.4(0.92) + 0.6(0.87)\right)\left(190,000\right) = 180,100$$

b. (10 points) The $Var[Y] = 793,000,000$ to the nearest 1,000,000. Calculate the $Var[Y]$ to the nearest 1000.

Solution:

$$E(Y^2) = 100,000^2(0.11) + 190,000^2(0.89) = 3.3229 * 10^{10}$$

$$Var(Y) = E(Y^2) - \left(E(Y)\right)^2 = 792,990,000$$

c. (15 points) John Life Insurance Company sells 10,000 annuities to Purdue University to independent lives all age x. Assuming the normal distribution, calculate the premium for the 10,000 policies that the John must charge Purdue to be 95% certain that the premium will be greater than the present value of benefits paid.

Solution:

$$E(S) = 10,000(180,100)$$

$$Var(S) = 10,000(792,990,000)$$

$$\sqrt{Var(S)} = 2,816,007.812$$

$$\Pr(PVBen < premium) = \Pr\left(z < \frac{premium - 180,100(10,000)}{\sqrt{Var(S)}}\right) = 0.95$$

$$\Pr\left(z < \frac{premium - 180,100(10,000)}{2,816,007.812}\right) = 0.95$$

$$\frac{premium - 180,100(10,000)}{2,816,007.812} = 1.645$$

$$premium = 180,563.23$$

3. For a two year endowment insurance policy on (x), you are given the follow profits (profit vector) and premiums for each policy that is in force at the start of the policy year:

Time (t)	Profit Vector	Premium at t-1	q_{x+t-1}	Withdrawal Rate at t
0	-500			
1	350	500	0.05	0.20
2	400	500	0.08	0.00

a. (5 points) Calculate the internal rate of return for this policy.

Solution:

$$CF_0 = -500$$
$$CF_1 = 350$$
$$CF_2 = 304$$
$$IRRCPT = 20.4693$$

Or

$$-500 + 350v + 304v^2 = 0 ==> v = \frac{-350 \pm \sqrt{350^2 - (4)(-500)(304)}}{(2)(304)} = 0.8300087052$$

$$i = v^{-1} - 1 = 0.2046929$$

b. (5 points) Calculate the Net Present Value of the profits using a 15% interest rate.

Solution:

$$NPV = -500 + \frac{350}{1.15} + \frac{304}{1.15^2} = 34.2155$$

c. (5 points) Calculate the profit margin for this policy using a 15% interest rate.

Solution:

$$PM = \frac{NPV}{PV(premiums)} = \frac{34.2155}{500 + \frac{500(0.95)(0.8)}{1.15}} = 0.0412$$

4. (15 points) A fully discrete whole life on (50) provides a death benefit of 100,000. You are given:

   i. The gross premium paid annually is 2400.
   ii. The reserve at the end of the 9th year is 15,000.
   iii. The reserve at the end of the 10th year is 16,200.
   iv. The reserve at the end of the 11th year is 17,500.
   v. Cash values are 85% of the reserves
   vi. Mortality follows the Illustrative Life Table.
   vii. Withdrawals are 15% in the first year and 4% all years thereafter.
   viii. The interest rate earned is 8% in all years.
   ix. Issue expenses are 50% of premium.
   x. Maintenance expenses are 7% of premium plus 60 per policy in all years including the first year.

Calculate the profit in the 10th year.

Solution:

$$profit = ({}_9V + P - Exp)(1+i) - Ben - {}_{10}V \cdot p_x - Withdrawals$$

$$Exp = 0.09(2400) + 60 = 228$$

$$Ben = 100,000 q_{59} = 1262$$

$$p_x = 1 - 0.01262 = 0.98738$$

$$profit = (15,000 + 2400 - 228)(1.08) - 1262 - 16,200(0.98738)(0.96) - (16,200)(0.98738)(0.85)(0.04)$$

$$= 1384.18$$