STAT 490
Quiz 2
Spring 2016
March 8, 2016

1. A life insurance company sells a whole life policy to (95) with a death benefit of 1,000,000.

   You are given that $q_{95} = 0.40$, $q_{96} = 0.70$ and $q_{97} = 1.00$.

   You are also given that the current spot interest rate curve:

Time t	y_t
1	0.04
2	0.05
3	0.06
4	0.07

   Premiums are paid annually.

   (4 points) The net benefit premium is 527,000 to the nearest 1000. Calculate the net benefit premium to the nearest 1.

   Solution:

   Let's calculate $l's.$ $l_{95} = 100; l_{96} = 100(1-0.4) = 60; l_{97} = 60(1-0.7) = 18; l_{98} = 0$

   $$P[100 + 60(1.04)^{-1} + 18(1.05)^{-2}] = (1,000,000)[40(1.04)^{-1} + 42(1.05)^{-2} + 18(1.06)^{-3}]$$

   $$\therefore P = 526,781.61$$

   (7 points) Calculate the net benefit reserve at the end of one year and at the end of two years.

   Solution:

This can be done using the recursive formula or PVFB-PVFP. Either way, you must use forward interest rates when applicable.

$${}_0V = 0$$

$${}_1V = \frac{({}_0V + P)(1 + f(0,1)) - (S)(q_x)}{1 - q_x} = \frac{(0 + 526,781.61)(1.04) - (1,000,000)(0.4)}{1 - 0.4} = 246,421.46$$

$${}_2V = \frac{({}_1V + P)(1 + f(1,2)) - (S)(q_{x+1})}{1 - q_{x+1}} \qquad \boxed{1 + f(1,2) = \frac{(1.05)^2}{1.04} = 1.060096154}$$

$$= \frac{(246,421.46 + 526,781.61)(1.060096154) - (1,000,000)(0.7)}{1 - 0.7} = 398,898.66$$

Or

$${}_1V = 1,000,000\left[\frac{42}{60}(1 + f(1,2))^{-1} + \frac{18}{60}(1 + f(1,3))^{-2}\right] - (526,781.61)\left[1 + \frac{18}{60}(1 + f(1,2))^{-1}\right]$$

$$\boxed{1 + f(1,3) = \left[\frac{(1.06)^3}{1.04}\right]^{1/2} = 1.070143772}$$

$$= 1,000,000\left[\frac{42}{60}(1.060096154)^{-1} + \frac{18}{60}(1.070143772)^{-2}\right] - (526,781.61)\left[1 + \frac{18}{60}(1.060096154)^{-1}\right]$$

$$= 246,421.45$$

$${}_2V = 1,000,000(1 + f(2,3))^{-1} - 526,781.61$$

$$\boxed{1 + f(2,3) = \left[\frac{(1.06)^3}{(1.05)^2}\right] = 1.080286621}$$

$$= 1,000,000(1.080286621)^{-1} - 526,781.61$$

$$= 398,898.65$$

(10 points) Let L_0^n be the loss at issue random variable for this insurance. As we discussed in class, since we are using spot interest rates, we cannot calculate the $Var[L_0^n]$ using the standard formula of $Var[L_0^n] = \left(S + \frac{P}{d}\right)^2 \left({}^2A_x - [A_x]^2\right)$. We must calculate it from first principles. Calculate the $Var[L_0^n]$.

Solution:

Event	**Present Value**	**Probability**
Die Year 1	$(1{,}000{,}000)(1.04)^{-1}$-526,781.61=434,756.85	0.40
Die Year 2	$(1{,}000{,}000)(1.05)^{-2}$-526,781.61$(1+1.04^{-1})$=-126,272.91	0.42
Die Year 3	$(1{,}000{,}000)(1.06)^{-3}$-526,781.61$(1+1.04^{-1}+1.05^{-2})$=-671,489.55	0.18

$$Var[L_0^n] = E[L_0^{n2}] - \left(E[L_0^n]\right)^2 = (434,756.85)^2(0.4) + (-126,272.91)^2(0.42) + (-671,489.55)^2(0.18) - 0$$

$$= 163,463,922,400 = (404,306.72)^2$$