Math 373
Spring 2013
Homework Solutions– Chapter 1

Non-Interest Theory

1. 2000 + 2003 + … + 3500 =

   Sum of an arithmetic series= $\mathbf{\dfrac{(First\ Term) + (Last\ Term)}{2}}$ **(# of Terms)**

   $$\frac{2000+3500}{2}*(501)=1,377,750$$

   **How do we know there are 501 terms? We know it takes 500 terms to get from 2000 to 3500 by 3's (3500-2000)*(1/3) = 500 but we have to include the "2000" term as well. This gives us 501 terms total.

2. 4 + 8 + 16 + 32 + . . . + 1024 =

   This is a geometric series. Each term is 2 multiplied by the prior term. Therefore,

   Sum of a geometric series= $\mathbf{\dfrac{(First\ Term) - (Next\ Term\ After\ the\ Last\ Term)}{1 - Ratio}}$

   $$\frac{4-(1024)(2)}{1-2}=2044$$

3. $1 + 0.9 + 0.9^2 + . . . 0.9^{12}$ =

   This is a sum of a geometric series.

   $$\frac{1-0.9^{13}}{1-0.9}=7.4581$$

4. If $(1+i)^5=1.1$, calculate $1+(1+i)^5+(1+i)^{10}+...+(1+i)^{100}$.

   Rewrite this by substituting $(1+i)^5$ with 1.1.

   $$1+1.1+1.1^2+...+1.1^{20}$$
   $$=\frac{1-1.1^{21}}{1-1.1}=64.0025$$

Chapter 1, Section 3

5. Ben borrows 5000 from Jeff. At the end of three years, Ben repays the loan with a payment of 5750.

   Calculate:

   a. The principal K for the loan.

      Principal= Amount Borrowed= 5000

   b. $A(0)$

      A(0)= Accumulated Value at t=0; =5000

   c. $A(3)$

      A(3)= Accumulated Value at t=3; =5750

   d. $a(3)$

      a(3)= amount $1 will accumulate to in 3 years

      $$\frac{A(3)}{A(0)} = \frac{5750}{5000} = 1.15$$

   e. The amount of interest paid by Ben

      Interest= S-K= 5750-5000= 750

   f. $i_{[0,3]}$

      $$i_{[0,3]} = \frac{a(3) - a(0)}{a(0)} = \frac{1.15 - 1}{1} = 0.15 = 15\%$$

6. You are given that $a(t) = 1 + 0.02t + 0.004t^2$.

Calculate:

a. If Matt invests 2500 now, how much will he have after 5 years?

$$\begin{aligned}&= 2500a(5)\\&= 2500(1 + 0.02(5) + 0.004(5^2))\\&= 3000\end{aligned}$$

b. Calculate i_8.

$$i_n = i_{[n-1,n]} = \frac{a(n) - a(n-1)}{a(n-1)}$$

$$i_8 = i_{[7,8]} = \frac{a(8) - a(7)}{a(7)}$$

$$a(8) = 1 + 0.02(8) + 0.004(8^2) = 1.416$$

$$a(7) = 1 + 0.02(7) + 0.004(7^2) = 1.336$$

$$i_8 = \frac{1.416 - 1.336}{1.336} = 0.059880 = 5.9880\%$$

c. Calculate the amount of interest that would be earned during the 3rd year if Eric invested 4000 today.

Note: Amount of interest earned during the 3rd year= a(3)-a(2)

$$4000\left[a(3) - a(2)\right]$$

$$a(3) = 1 + 0.02(3) + 0.004(3^2) = 1.096$$

$$a(2) = 1 + 0.02(2) + 0.004(2^2) = 1.056$$

$$4000\left[a(3) - a(2)\right] = 4000\left[1.096 - 1.056\right] = 160$$

7. You are given:

   i. $i_1 = 0.05$

   ii. $i_2 = 0.04$

   iii. $i_3 = 0.06$

   Calculate $a(3)$.

First, write down the equations that we know.

$$i_1 = \frac{a(1)-a(0)}{a(0)}; i_2 = \frac{a(2)-a(1)}{a(1)}; i_3 = \frac{a(3)-a(2)}{a(2)};$$

We also know that $a(1) = 1 + i_1 = 1.05$.

Using substitution into the second equation above we can solve for $a(2)$.

$$i_2 = \frac{a(2)-a(1)}{a(1)}$$

$$0.04 = \frac{a(2)-1.05}{1.05};$$

$$0.04(1.05) + 1.05 = a(2)$$

$$a(2) = 1.092$$

Finally, we can substitute into the third equation above to find $a(3)$.

$$i_3 = \frac{a(3)-a(2)}{a(2)}$$

$$0.06 = \frac{a(3)-1.092}{1.092}$$

$$0.06(1.092) + 1.092 = a(3)$$

$$a(3) = 1.15752$$

8. Kehara invests 10,000 in and account that has an accumulation function of $\alpha+\beta t+\delta t^2$. At the end of one year Kehara has 11,200 in his account. At the end of 2 years, Kehara has 13,000.

Determine α, β, and δ.

$$10,000a(1)=11,200$$
$$10,000a(2)=13,000$$
$$\rightarrow a(1)=1.12$$
$$\rightarrow a(2)=1.3$$

We are given that

$$a(t)=\alpha+\beta t+\delta t^2$$
$$\rightarrow a(1)=\alpha+\beta+\delta=1.12$$
$$\rightarrow a(2)=\alpha+2\beta+4\delta=1.3$$

We also know that

$$a(0)=1=\alpha+\beta*0+\delta*0$$
$$\rightarrow \alpha=1$$

This gives us two equations with two unknowns:

1. $1.12=1+\beta+\delta$
2. $1.3=1+2\beta+4\delta$

We can solve these equations using substitution or linear combinations. I will use substitution.

$$1.12=1+\beta+\delta;\rightarrow \beta=0.12-\delta$$
$$1.3=1+2\beta+4\delta;\rightarrow 1.3=1+2(0.12-\delta)+4\delta$$
$$\rightarrow 1.3=1.24-2\delta+4\delta$$
$$\rightarrow 0.06=2\delta$$
$$\delta=0.03$$

Now, plug this value into an equation to find β.

$$1.12=1+\beta+0.03$$
$$\beta=0.09$$

9. Poppy has a choice of two investments. Investment 1 has an accumulation function of $a(t)=1+0.001t^3$. Investment 2 has an accumulation function of $a(t)=1+0.01t^2$.

   a. Which investment should Poppy use if Poppy will be investing for 5 years? Explain why.
      Investment 1: $a(5)=1+0.001(5^3)=1.125$
      Investment 2: $a(5)=1+0.01(5^2)=1.25$
      Since 1.25>1.125, we should invest in **Investment 2**, because a dollar will accumulate to a great amount over the 5 year period.

   b. Which investment should Poppy use if Poppy will be investing for 15 years? Explain why.
      Investment 1: $a(15)=1+0.001(15^3)=4.375$
      Investment 2: $a(15)=1+0.01(15)^2=3.25$
      Since 4.375>3.25, we should invest in **Investment 1**, because a dollar will accumulate to a great amount over the 5 year period.

   c. If Poppy invests 100 into both investments, at what time will the accumulated value of both investments be equal?

      Let $a_1(t)$ be the accumulation function for investment 1 and $a_2(t)$ be the accumulation function for investment 2.

      $100a_1(t)=100a_2(t);\rightarrow a_1(t)=a_2(t)$
      $1+0.001t^3=1+0.01t^2;\rightarrow 0.001t^3=0.01t^2$
      $0.001t^3-0.01t^2=0$
      $t^2(0.001t-0.01)=0$
      This will give us two solutions.
      $t^2=0;\rightarrow t=0$ OR $0.001t-0.1=0;\rightarrow t=10$
      Our selected answer is **10 years.**

Chapter 1, Section 4

10. Jake invests 2800 in an account earning simple interest at an annual rate of 9%.

    Calculate:

    a. The amount of interest that Jake will earn in the first year.

    $= A_{2800}(1) - A_{2800}(0)$

    $A_{2800}(0) = 2800$

    $A_{2800}(1) = 2800(1 + 0.09) = 3052$

    $3052 - 2800 = 252$

    b. The amount of interest that Jake will earn in the 15th year.

    $= A_{2800}(15) - A_{2800}(14)$

    $A_{2800}(14) = 2800\left[1 + 0.09(14)\right] = 6328$

    $A_{2800}(15) = 2800\left[1 + 0.09(15)\right] = 6580$

    $6580 - 6328 = 252$

    NOTE: with simple interest, the same amount of interest in earned in each period.

    c. How much money Jake will have after 20 years?

    $A_{2800}(20) = 2800\left[1 + 0.09(20)\right] = 7840$

    d. The time t such that Jake's investment will double. (Note: t does not need to be an integer.)

    $A_{2800}(t) = 2800(2) = 5600 = 2800\left[1 + 0.09t\right]$

    $2 = 1 + 0.09t$

    $1 = 0.09t; \rightarrow t = 11.1111 \; years$

11. Victoria invests 1000 in an account that pays monthly simple interest at a rate of 0.6%.

Calculate:

a. The annual simple interest rate earned by Victoria.

The annual simple interest rate earned by Victoria is simply the monthly rate multiplied by 12. Annual rate = (0.006)(12) = 0.072 = 7.2%

b. The amount that Victoria will have after 18 months.

Using the monthly rate => $1000[1+(0.006)(18)]=1108$

Using the annual rate => $1000[1+(0.072)(1.5)]=1108$

c. The amount that Victoria will have after 2 years.

Using the monthly rate => $1000[1+(0.006)(24)]=1144$

Using the annual rate => $1000[1+(0.072)(2)]=1144$

12. Ben borrows 5000 from Jeff. Three years later, Ben repays the loan with a payment of 5750.

Assuming that Ben is paying simple interest, calculate the annual simple interest rate that he is paying on the loan.

$$5000\left[1+3s\right]=5750$$
$$\left[1+3s\right]=1.15$$
$$3s=0.15;\rightarrow s=0.05=5\%$$

13. Heather borrows K to buy a new car. The loan has simple interest of 7%. At the end of 5 years, Heather repays the loan with a payment of 16,605.

Determine K.

$$K\left[1+0.07(5)\right]=16,605$$
$$K=\frac{16,605}{1.35}=12,300$$

14. Mengyang lends Yingcong 1000 at a simple interest rate of 15%. What will be the effective interest rate on the loan during year 10.

K=1000 s=15%

$$i_{[9,10]} = \frac{a(10) - a(9)}{a(9)}$$

$$a(10) = 1 + 10(0.15) = 2.50; a(9) = 1 + 9(0.15) = 2.35$$

$$i_{[9,10]} = \frac{2.50 - 2.35}{2.35} = 0.063830 = 6.3830\%$$

15. Jayme loans Laura 20,000 at a simple interest rate of $s\%$. During the 5th year of the loan, Jayme earns an effective interest rate of 4%. Colleen repays the loan after 10.5 years.

Calculate the amount that Laura has to pay.

$$i_{[4,5]} = 4\%; K = 20000; n = 10.5$$

$$A_{20000}(10.5) = 20000\left[1 + 10.5s\right]$$

We can use $i_{[4,5]}$ to find s.

$$i_{[4,5]} = \frac{a(5) - a(4)}{a(4)}$$

$$a(5) = 1 + 5s; a(4) = 1 + 4s$$

$$i_{[4,5]} = \frac{1 + 5s - 1 - 4s}{1 + 4s} = \frac{s}{1 + 4s} = 0.04$$

$$0.04 + 0.16s = s$$

$$s = 0.047619$$

$$A_{20000}(10.5) = 20000\left[1 + (10.5)(0.047619)\right] = 30000$$

16. Zexi invests in fund earning simple interest of 10%.

The effective interest rate for the n^th^ year (i_n) is 5%.

Calculate n.

$$s = 10\%; i_n = 5\%$$
$$i_n = \frac{a(n) - a(n-1)}{a(n-1)}$$
$$a(n) = 1 + 0.1n$$
$$a(n-1) = 1 + 0.1(n-1) = 0.9 + 0.1n$$
$$i_n = 0.05 = \frac{1 + 0.1n - 0.9 - 0.1n}{0.9 + 0.1n} = \frac{0.1}{0.9 + 0.1n} = 0.05$$
$$0.045 + 0.005n = 0.1$$
$$0.005n = 0.055; \to n = 11$$

Chapter 1, Section 5

17. Jake invests 2800 in an account earning compound interest at an annual rate of 9%.

    Calculate:

    a. The amount of interest that Jake will earn in the first year.

       $A_{2800}(1) - A_{2800}(0)$

       $2800(1.09) - 2800 = 252$

    b. The amount of interest that Jake will earn in the 15th year.

       $A_{2800}(15) - A_{2800}(14)$

       $2800(1.09)^{15} - 2800(1.09)^{14} = 842.12$

       NOTE: The interest earned in the first period is the same regardless if we are using simple or compound interest. But, after the first period the interest earned is different depending on the method of interest accumulation.

    c. How much money Jake will have after 20 years?

       $A_{2800}(20) = 2800(1.09)^{20} = 15,692.35$

    d. The time t such that Zach's investment will double. (Note: t does not need to be an integer.)

       $A_{2800}(t) = 2800(2) = 2800(1.09)^t$

       $2 = \left(1.09\right)^t$

       $t = \dfrac{\ln(2)}{\ln(1.09)} = 8.0432\,years$

18. Ben borrows 5000 from Jeff. Three years later, Ben repays the loan with a payment of 5750.

    Assuming that Ben is paying compound interest, calculate the annual compound interest rate that he is paying on the loan.

    $5000a(3) = 5750$

    $a(3) = 1.15$

    $(1+i)^3 = 1.15$

    $i = \sqrt[3]{1.15} - 1 = 0.0476895 = 4.4690\%$

19. Xuening borrows 1000 at an annual compound interest rate of 4.35%. Xuening repays the loan at the end of 6.5 years.

    Calculate the amount the Xuening must pay to repay the loan.

    $$A_{1000}(6.5) = 1000(1.0435)^{6.5} = 1318.87$$

20. Rose invests money in a bank account earning compound interest at an annual effective interest rate of 5%.

    Tim invests money in a bank account earning 10% simple interest.

    What year will Rose and Tim earn the same annual effective interest rate?

    For simple interest: $i_n = \dfrac{s}{a+s(n-1)}$

    For compound interest: $i_n = (1+i)-1 = i$

    $$\begin{aligned} &s = 0.1; i = 0.05 \\ &i_n = 0.05 = \frac{0.1}{1+0.1(n-1)} = \frac{0.1}{1+0.1n-0.1} = \frac{0.1}{0.9+0.1n} \\ &0.045+0.005n = 0.1 \\ &0.005n = 0.055 \\ &n = 11 \end{aligned}$$

21. Yuxi invests 1000 for a period of 9 years in an account earning compound interest. During the first 4 years, the account earns 5%. During the next 3 years, the account earns 7%. During the last 2 years, the account earns 4%.

    Ethan invests 1000 for a period of 9 years in an account earning compound interest. Ethan's interest rate is level for all 9 years.

    At the end of 9 years, Yuxi and Ethan have the same amount in their investment account. Determine the level annual interest rate earned by Ethan.

    Yuxi: $A_{1000}(9) = 1000(1.05)^4(1.07)^3(1.04)^2 = 1610.553693$

    Ethan: $\begin{aligned} &A_{1000}(9) = 1610.553693 = 1000(1+i)^9 \\ &i = \sqrt[9]{1.610553693} - 1 = 0.054380 = 5.4380\% \end{aligned}$

22. Jacob invests 1000 in an account earning simple interest of 4%.

Cassie invests X in an account earning 2% compounded annually.

In year Y, Cassie and Jacob earn the same annual effective interest rate.

At the end of year Y, the amount of money in Cassie's account is equal to the amount of money in Jacob's account.

Determine X.

Solution:
First find Y

$$i = \frac{s}{1+s(Y-1)} = 0.02 = \frac{0.04}{1+0.04(Y-1)} = \frac{0.04}{0.96+0.04Y}$$

$$0.0192 + 0.0008Y = 0.04$$

$$0.0208 = 0.0008Y; \rightarrow Y = 26$$

Now we can find X

$$X(1.02)^Y = 1000(1+0.04Y)$$

$$X(1.02)^{26} = 1000(1+.04*26)$$

$$X(1.02)^{26} = 2040; \rightarrow X = 1219.06$$

23. Alex invests 100 in an account earning simple interest. During the 10th year, the amount of interest that Alex earns is 20.

Taylor invests X into an account earning compound interest of i. During the 10th year, the amount of interest that Taylor earns is also 20.

During the 11th year, Alex and Taylor earn the same annual effective interest rate.

Determine X.

Solution

Amount of Interest in the 10th year = $A(10)-A(9)=K\cdot a(10)-K\cdot a(9)=K[a(10)-a(9)]$

For Alex with simple interest

Amount of interest in 10th year =

$$20=K[a(10)-a(9)]=100[(1+s\cdot 10)-(1+s\cdot 9)]=100\cdot s \text{ ==> } s=\frac{20}{100}=0.20$$

Annual effective interest rate in 11th year for Alex under simple interest =

$$i_n=\frac{s}{1+(n-1)s}=i_{11}=\frac{0.2}{1+(11-1)0.2}=\frac{0.2}{3}=0.066667$$

Annual effective interest rate for 11th year for Taylor under compound interest = $i_n=i=i_{11}$

Since i_{11} for Alex = i_{11} for Taylor, $i=0.066667$

For Taylor with compound interest

Amount of interest in 10th year =

$$20=K[a(10)-a(9)]=X[(1+i)^{10}-(1+i)^{9}]=X[(1.066667)^{10}-(1.066667)^{9}]$$

Therefore, $X=\dfrac{20}{(1.066667)^{10}-(1.066667)^{9}}=167.82$

Chapter 1, Section 6

24. Hao borrows 20,000 to buy a car and pays interest in advance at a discount rate of 6%. Calculate the amount of discount and the amount of Hao has to spend on a car.

    Solution:

    Amount of Discount= KD=20000(.06)=1200
    Amount Xuan has to spend = K-KD=20000-1200=18800

25. Sarah borrows 12,000 to buy a new car and pays 900 of interest in advance. The period of the loan is one year.
    a. Calculate $d_{[0,1]}$.

       Solution:

       $$d_{[0,1]} = \frac{Adv.Interest}{Amt.Borrowed} = \frac{900}{12000} = 7.50\%$$

    b. Calculate the annual effective interest rate for this loan.

       Solution:

       $i =$ annual effective interest rate
       $d =$ discount rate

       $$d = \frac{i}{1+i} \rightarrow d(1+i) = i = d + id$$
       $$i - id = d \rightarrow i(1-d) = d$$
       $$i = \frac{d}{1-d} = \frac{0.075}{1-0.075} = 0.081081 = 8.1081\%$$

26. You are given that $a(t) = 1 + 0.03t + 0.005t^2$. Calculate d_8.

Remember that $i_8 = \frac{d_8}{1-d_8}$. We can find i_8 using $\frac{a(8)-a(7)}{a(7)}$.

$$a(8) = 1 + 0.03(8) + 0.005(8^2) = 1.560$$

$$a(7) = 1 + 0.03(7) + 0.005(7^2) = 1.455$$

$$i_8 = \frac{1.560 - 1.455}{1.455} = 0.07216495$$

Now we can find d_8.

$$0.072165 = \frac{d_8}{1-d_8}$$

$$d_8 = \frac{0.072165}{1.072165} = 0.067308 = 6.7308\%$$

Chapter 1, Section 7

27. You are given $a(t) = 1 + 0.03t + 0.005t^2$. Calculate $v(7)$.

Solution:

We must remember that $v(t) = \frac{1}{a(t)}$. Knowing this fact makes this problem very simple.

$$a(7) = 1 + 0.03(7) + 0.005(7^2) = 1.455$$

$$v(7) = \frac{1}{1.455} = 0.687285$$

28. An investment account earns 5% simple interest. Calculate the amount the Deanna must invest today to have 10,000 in 8 years.

Solution:

There are two ways to set up this problem.

$10000v(8) = x$ or $xa(8) = 10000$. It is very important to realize that these two equations are equivalent! You may choose which way you want to write the problem. The calculations are the same.

$$v(8) = \frac{1}{(1 + 0.05(8))}$$

$$10000v(8) = 7142.86$$

29. An investment account earns 5% compound interest. Calculate the amount the Deanna must invest today to have 10,000 in 8 years.

Solution:

Once again we could write this problem in two ways.

$10000v(8) = x$ or $xa(8) = 10000$. Also, note that we are using the same equations as we did in 28. The only difference is that now our $v(8)$ and $a(8)$ will be formulated differently because we are using compound interest.

$$v(8) = \frac{1}{(1.05)^8}$$

$$10000v(8) = 6768.39$$

30. You are given that $v(t) = \dfrac{1}{(\alpha t + \beta)}$.

Using this discount function, a payment of 200 to be made at time 10 has a present value of 100 at time 0.

Calculate $a(15)$.

Solution:

First, since $v(t) = \dfrac{1}{a(t)}$

Then, $a(t) = \alpha t + \beta$.

We are given that $100a(10) = 200$, therefore, $a(10) = 2$.

We also know by definition that $a(0) = 1 \Rightarrow \alpha(0) + \beta = 1 \Rightarrow \beta = 1$

Then

$$a(10) = 2 = \alpha(10) + \beta = \alpha(10) + 1$$

$$1 = 10\alpha$$

$$\alpha = 10\%$$

$$a(15) = 1 + 15(0.1) = 2.5$$

31. Ryan wants to buy a car for 30,000 at the end of 10 years. He plans to invest an amount of D at the end of 2 years so it will grow to 30,000 at the end of 10 years. If Ryan's account will earn 10% compound interest, determine the amount that Ryan must deposit.

Solution:

$$30000v(8) = D$$

Using compound interest: $30000\left(\frac{1}{1.10^8}\right) = 13995.22$

How much more would he have to deposit if the account earned simple interest instead?
Solution:

Using simple interest:

Using the ratio approach from class

$$\frac{a(2)}{a(10)} = \frac{X}{30,000} ==> X = (30,000)\left(\frac{a(2)}{a(10)}\right) = 30,000\left(\frac{1+(0.1)(2)}{1+(0.1)(10)}\right) = 18,000$$

$$18,000.00 - 13995.22 = 4004.78$$

32. Pianpian must invest 1000 now in an account earning compound interest to have 2000 after Y years. How much must Pianpian invest now to have 2000 after $2Y$ years.

Solution:

We know that $1000a(Y) = 2000; \rightarrow a(Y) = 2$

Since Pianpian is earning compound interest, $a(Y) = 2 = (1+i)^Y$. We want to find $a(2Y) = (1+i)^{2Y}$.

This can be rewritten as $a(2Y) = \left((1+i)^Y\right)^2$. Now from what we know we can substitute.

$$(1+i)^Y = 2$$
$$\left((1+i)^Y\right)^2 = 2^2 = 4$$

To find how much Pianpian needs to invest,

$$xa(2Y) = 2000$$
$$x(4) = 2000$$
$$x = 500$$

33. The present value of 9,285 payable at the end of 14 years assuming compound interest at an annual effective rate of i is 6,125.

Determine i.

Solution:

$$6125a(14) = 9285$$

$$a(14) = 1.5159184 = (1+i)^{14}$$

$$\sqrt[14]{1.5159184} - 1 = i = 0.030162 = 3.0162\%$$

34. The accumulated value of $1 invested 2 years ago less the present value of $1 to be paid in two years is equal to (4641/12100). Calculate i.

Solution:

FV of $1 invested 2 years ago= $1(1+i)^2$

PV of $1 to be paid in 2 years= $1(1+i)^{-2}$

From the problem, we can write the equation: $(1+i)^2-(1+i)^{-2}=\frac{4641}{12100}$.

Multiply through by 12100; $12100(1+i)^2-12100(1+i)^{-2}=4641$.

To make this equation look simpler, substitute $x=(1+i)^2$.

$12100x^2-12100x^{-2}=4641$

Now, multiply through by x^2: $12100x^4-12100=4641x^2$

Rewrite: $12100x^4-4641x^2-12100=0$

Finally, if we substitute $y=x^2$ we get a simple quadratic equation that we know how to solve.

$12100y^2-4641y-12100=0$

$$y=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$y=\frac{4641\pm\sqrt{4641^2-4(12100)(-12100)}}{2(12100)}=\frac{4641\pm 24641}{24200}$$

This will yield two solutions:

$$y=\frac{4641+24641}{24200}=1.21 \text{ OR } y=\frac{4641-24641}{24200}=-0.826$$

The second solution is unreasonable because it is negative.

Using the first solution, we can reverse our substitutions to find our final answers.

$y=x^2=(1+i)^2=1.21$

$i=\sqrt{1.21}-1=0.1=10\%$

35. Wang Corporation is building a new factory. The factory is expected to generate the following cash flows:

Time	Cash Flow
0	-200
1	-30
2	90
3	120
4	160

Calculate the net present value of this project at 15%. Calculate the internal rate of return.

Solution:

There are two methods of solving for the NPV: by hand or using the financial calculator.

By hand:

$$NPV = CF_0 + CF_1 v + CF_2 v^2 + CF_3 v^3 + CF_4 v^4$$

$$NPV = -200 - 30(1.15)^{-1} + 90(1.15)^{-2} + 120(1.15)^{-3} + 160(1.15)^{-4} = 12.3484$$

You will find that it is much easier to calculate NPV using a calculator, especially because in order to solve for the IRR you will need to use a financial calculator or excel.

Using Calculator:

$$\boxed{CF_0 = -200};\ \boxed{C01 = -30};\ \boxed{F01 = 1};\ \boxed{C02 = 90};\ \boxed{F02 = 1};$$

$$\boxed{C03 = 120};\ \boxed{F03 = 1};\ \boxed{C04 = 160};\ \boxed{F04 = 1};$$

$$\boxed{NPV};\ \boxed{I = 15}; \Rightarrow NPV = 12.3484$$

$$\boxed{CPT}\boxed{IRR} = 17.0721\%$$

36. Kendrick Corporation invests X million today to build a factory. The factory is expected to produce the following profits:

End of Year	Profits
1	1 million
2	4 million
3	2 million
4	1 million

At the end of 4 years, the factory will be obsolete and will be closed.

The Net Present Value of this project to Kendrick Corporation is 1 million at an interest rate of 8%.

Calculate the Internal Rate of Return on this project.

Solution:

For this problem, we need to do part of it by hand first, then we can use the financial calculator.

By hand: $$NPV = 1 = -x + 1(1.08)^{-1} + 4(1.08)^{-2} + 2(1.08)^{-3} + 1(1.08)^{-1}$$
$$x = 5.677976 million$$

Using calculator:

$\boxed{CF_0 = -5.677967}$; $\boxed{C01 = 1}$; $\boxed{F01 = 1}$; $\boxed{C02 = 4}$; $\boxed{F02 = 1}$;

$\boxed{C03 = 2}$; $\boxed{F03 = 1}$; $\boxed{C04 = 1}$; $\boxed{F04 = 1}$;

$\boxed{CPT}\boxed{IRR} = 15.913\%$

Chapter 1, Section 1.9

37. Scott borrows 20,000. The loan is repaid in 5 years at an annual rate of discount of 7%.

Calculate:

a. The amount of cash that Scott will receive at time 0.

Solution:

$$20000 = x(1-d)^{-5} = x(1-0.07)^{-5}$$
$$x = \frac{20000}{0.93^{-5}} = 13913.77$$

b. The amount of discount that Scott will pay.

Solution:

Amount of discount= Total amount of loan—amount received at time 0
$$= 20000 - 13913.77 = 6086.23$$

c. The equivalent compound annual interest rate.

Solution:

$$(1-d)^{-1} = (1+i)$$
$$i = \frac{1}{1-d} - 1 = \frac{1}{0.93} - 1 = 7.5269\%$$

38. Sam invests 12,000 at a rate that is equivalent to an annual discount rate of 4%. Calculate the amount that Sam will have at the end of 7 years.

Solution:
$$12000(1+i)^7 = 12000(1-d)^{-7} = 12000(0.96)^{-7} = 15969.18$$

39. Victoria wants to buy a new piano for 8,000. The loan is for 3 years at an annual discount rate of 8%. Determine the amount that Victoria must borrow to finance the entire purchase price of the piano.

Solution:

$$8000(1-d)^{-3} = 8000(0.92)^{-3} = 10273.69$$

40. Telma has a choice between two three year loans. Loan A is subject to compound interest while Loan B is subject to compound discount. The annual interest rate on Loan A is equivalent to the annual discount rate on Loan B.

If Loan A is 2000, the amount of interest on that Telma will pay on Loan A is 662.

Calculate the amount of discount that Telma will pay on Loan B if the amount of Loan B is 1000. (Note: Under discount, the amount of the loan is the amount that must be repaid at the end of the loan, not the amount of cash received at the start of the loan.)

Solution:

We're given that the amount of the interest paid on loan A is 662. This means that the amount paid back minus the loan amount must be 662

$$2000(1+i)^3 - 2000 = 662$$
$$i = \left(\frac{2662}{2000}\right)^{1/3} - 1 = 0.1 = 10\%$$

Before moving on to loan B, let's find the equivalent discount rate:

$$(1+i) = (1-d)^{-1} \rightarrow d = 1-(1.1)^{-1} = \frac{1}{11}$$

Now, looking at loan B, we need to find the amount of discount. This value is equal to the amount of the loan less the amount reviewed at time zero.

To find the amount received at time zero, $1000 = x(1-d)^{-3}; \rightarrow x = \dfrac{1000}{\left(1-\frac{1}{11}\right)^{-3}} = 751.3148$

Finally the amount of discount is: $1000 - 751.3148 = 248.69$

41. Kevin invests 100 in a bank account earning a simple interest rate of 9%. James invests 100 in a bank account which earns an annual effective discount rate of d. At the end of 12 years, Kevin and James have the same amount of money in their accounts. Calculate d.

Solution:

Kevin: $FV = x = 100(1+st) = 100(1+0.09*12) = 208$

$FV = 208 = 100(1-d)^{-12}$

James: $2.08 = (1-d)^{-12}$;

$d = 1 - 2.08^{-1/12} = 5.9206\%$

42. Garret needs 1000 to buy a refrigerator for her apartment. The bank offers her the following three loans:

a. A loan that requires simple interest at an annual interest rate of 10%.

$$FV = 1000(1+.1*3) = 1300$$

b. A loan that requires compound interest at an annual interest rate of 9.2%.

$$FV = 1000(1.092)^3 = 1302.17$$

c. A loan with interest in advance at an annual rate of discount of 8.5%.

$$(1-d)^{-1} = (1+i)$$
$$(1+i) = (1-.085)^{-1}$$
$$FV = 1000(1-.085)^{-3} = 1305.379$$

Assuming that Garret repays the loan at the end of three years, determine which loan Garret should accept and explain why.

Solution: Garret should accept Loan A because it has the smallest future value. This means he will pay the least amount of interest if he accepts Loan A.

Chapter 1, Section 1.10

43. An account pays a nominal interest rate of 8% compounded monthly.
NOTE: The key to understanding the problems in this section is knowing the following relationship: $(1+i)=\left(1+\frac{i^{(m)}}{m}\right)^{m}=(1-d)^{-1}=\left(1-\frac{d^{(m)}}{m}\right)^{-m}$

a. Calculate the monthly effective interest rate.

Solution:

$$i^{(12)}=.08$$
$$\frac{i^{(12)}}{12}=.006667=0.6667\%$$

b. Calculate the annual effective interest rate.

Solution:

$$(1+i)=\left(1+\frac{i^{(12)}}{12}\right)^{12}$$
$$i=\left(1+\frac{0.08}{12}\right)^{12}-1=8.30\%$$

c. Calculate the annual effective discount rate.

Solution:

$$(1+i)=(1-d)^{-1}$$
$$1.0830=\frac{1}{1-d}$$
$$d=1-\frac{1}{1.0830}=0.0766=7.66\%$$

d. Calculate the quarterly effective interest rate.

Solution:

$$(1+i)=\left(1+\frac{i^{(4)}}{4}\right)^{4}$$
$$\left[(1.0830)^{\frac{1}{4}}-1\right]=\frac{i^{(4)}}{4}=0.020134=2.0134\%$$

44. An account pays an annual effective interest rate of 9%.

a. Calculate the monthly effective rate of interest.

Solution:

$$(1+i)=\left(1+\frac{i^{(12)}}{12}\right)^{12}$$

$$\left(1.09\right)^{\frac{1}{12}}-1=\frac{i^{(12)}}{12}=0.007207=0.7207\%$$

b. Calculate the $i^{(12)}$.

Solution:

$$i^{(12)}=12(.007207)=0.08649=8.649\%$$

c. Calculate $d^{(2)}$.

Solution:

$$\left(1-\frac{d^{(2)}}{2}\right)^{-2}=(1+i)$$

$$\left(1-(1.09)^{-\frac{1}{2}}\right)2=0.084347=8.4347\%$$

45. You are given $d^{(4)} = 0.06$. Calculate:

a. Calculate the quarterly effective discount rate.

Solution:

$$\frac{d^{(4)}}{4} = \frac{0.06}{4} = 0.015 = 1.50\%$$

b. Calculate the annual effective interest rate.

Solution:

$$(1+i) = \left(1 - \frac{d^4}{4}\right)^{-4}$$

$$i = (1-0.015)^{-4} - 1 = 0.062319 = 6.2319\%$$

c. Calculate the nominal interest rate compounded monthly

Solution:

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = (1+i)$$

$$\left[(1.062319)^{\frac{1}{12}} - 1\right]12 = i^{(12)} = 0.060607 = 6.0607\%$$

46. Siyun invests 6000 in an account that earns 8% compounded monthly. How much will Siyun have after 3 years.

Solution:

$$FV = 6000\left(1 + \frac{.08}{12}\right)^{12*3} = 7621.42$$

47. Amber invests 3000 in an account that earns a nominal interest rate of $i^{(4)}$. At the end of 8 years, Amber has 4100.

Calculate $i^{(4)}$.

Solution:

$$3000\left(1+\frac{i^{(4)}}{4}\right)^{4*8}=4100$$

$$\left(1+\frac{i^{(4)}}{4}\right)^{32}=1.366667$$

$$i^{(4)}=\left(1.366667^{\frac{1}{32}}-1\right)4=0.039238=3.9238\%$$

48. Brett deposits D into an account that earns 11% compounded semi-annually. At the end of 4 ½ years, Brett has 100,000.

Determine D.

Solution:

$$FV=100000=D\left(1+\frac{.11}{2}\right)^{2*4.5}$$

$$D=\frac{100000}{1.61909}=61762.93$$

49. Peter borrows 10,000 at an interest rate of 5% compounded monthly. At the end of the loan, Peter pays 15,474.23.

Calculate the length of Peter's loan in years.

Solution:

$$10000\left(1+\frac{.05}{12}\right)^{12N}=15474.23$$

$$\left(1+\frac{.05}{12}\right)^{12N}=1.547423$$

$$12N\ln\left(1+\frac{.05}{12}\right)=\ln(1.547423)$$

$$N=\frac{\ln(1.547423)}{\ln\left(1+\frac{.05}{12}\right)}*\frac{1}{12}=8.75\,years$$

50. Siyi borrows 10,000 for two years at a rate of discount of 6% convertible monthly. Calculate the amount of discount that Siyi will pay.

Solution:

Amount of Discount= Amount of Loan- Amount Received at t=0

$$PV=x=10000\left(1-\frac{0.06}{12}\right)^{2*12}=8866.535105$$

$$10000-8866.535105=1133.46$$

51. Yue invests 750 in an account that earns interest at a rate equivalent to a discount rate of 8% convertible quarterly. How much will Yue have after 15 months.

Solution:

$$FV=750\left(1-\frac{.08}{4}\right)^{-4*1.25}=829.72$$

52. Bo wants to borrow money to buy a car.

Bank A offers Bo a loan with an annual effective interest rate of 8.0%.

Bank B offers Bo a loan with an interest rate of 7.8% compounded monthly.

Which loan should Bo select and state why. (Provide work to support your answer.)

Solution:

One way to compare Bank A with Bank B is to find the equivalent annual effective interest rate for Bank B and compare it to 8.0%. Ultimately, Bo should choose the loan that has the lowest annual effective interest rate.

$$(1+i)=\left(1+\frac{.078}{12}\right)^{12}$$

$$i=0.08085=8.085\%$$

Since 8.085%>8.0%, Bo should accept the loan from Bank A.

53. Ravi invests 1000 in an account for nine years.

During the first three years, Ravi earns a quarterly effective interest rate of 2%.

During the next two years, Ravi earns $d^{(12)}=0.08$.

During the last four years, Ravi earns a nominal interest rate of 7% compounded monthly.

How much does Ravi have at the end of nine years?

Solution:

$$FV=1000(1+.02)^{4*3}\left(1-\frac{.08}{12}\right)^{-12*2}\left(1+\frac{.07}{12}\right)^{12*4}=1968.66$$

54. Megan invests 100,000 in an account that earns a nominal interest rate of 10% compounded every 4 years.

   a. Calculate the amount that Megan will have at the end of 4 years.

      Solution:

$$FV = 100000\left(1+\frac{.10}{.25}\right)^{4*.25} = 140000$$

      NOTE: Whenever the (m) in our formula is the number of times interest is compounded per year. Therefore, if interest is compounded less frequently than one year, our (m) will be less than one. In this case m=1/4 since it is compounded once every four years.

   b. Calculate the amount that Megan will have at the end of 7 years.

      Solution:

$$FV = 100000\left(1+\frac{.1}{.25}\right)^{.25(7)} = 180187.25$$

55. Boxi invests 10,000 in an account earning a nominal interest rate of 6% convertible every two years.

Zhengyi invests K in an account earning a nominal rate of interest of 6% convertible monthly.

After 10 years, the amount in Boxi's account is equal to the amount in Zhengyi's account.

Determine K.

Solution:

Boxi's Account:

$$FV = 10000\left(1+\frac{.06}{.5}\right)^{.5*10} = 17623.41683$$

Zjemgyi's Account:

$$FV = 17623.41683 = K\left(1+\frac{.06}{12}\right)^{12(10)}$$

$$K = 9686.41$$

Chapter 1, Section 1.11

NOTE: In this section we add on to our previous equation to include $(1+i)=e^{\delta}$

56. You are given that $i^{(4)} = 7\%$. Calculate δ.

Solution:

$$\left(1+\frac{.07}{4}\right)^{4} = e^{\delta} = 1.071859031$$

$$\delta = \ln(1.071859031) = 0.069395 = 6.9395\%$$

57. At age 20, Keith inherits 27,500. Keith invests his inheritance in a fund earning an interest rate of 9% convertible continuously. How much does Keith have at age 65.

Solution:

$$27,500e^{0.09(65-20)} = 1,578,430.07$$

58. Kyle is the beneficiary of a trust fund that will pay him 1 million at exact age 35. If Kyle is currently exact age 21, calculate the present value of Kyle's future payment using an interest rate of 7.2% compounded continuously.

Solution:

$$PV = 1000000e^{-0.072(35-21)} = 364948.15$$

Chapter 1, Section 1.12

59. You are given that $a(t) = 1 + 0.003t^2$. Calculate:

a. δ_t

Solution:

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{2(0.003)t}{1+0.003t^2} = \frac{0.006t}{1+0.003t^2}$$

b. δ_{10}

Solution:

$$\delta_{10} = \frac{0.006(10)}{1+0.003(10^2)} = 0.04615 = 4.615\%$$

60. You are given:

$$v(t) = \frac{1}{(1+0.02t)(1+.01t^2)}$$

Calculate δ_3.

Solution:

Remember that v(t)=1/a(t)

$$a(t) = (1+.02t)(1+.01t^2) = 0.0002t^3 + 0.01t^2 + 0.02t + 1$$

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{0.0006t^2 + 0.02t + 0.02}{0.0002t^3 + 0.01t^2 + 0.02t + 1}$$

$$\delta_3 = \frac{0.0006(3)^2 + 0.02(3) + 0.02}{0.0002(3)^3 + 0.01(3)^2 + 0.02(3) + 1} = 0.07391 = 7.391\%$$

61. Under simple interest with an interest rate of 5%, calculate δ_{20}.

Solution:

For simple interest, $a(t) = 1 + st$

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{s}{1+st}$$

$$\delta_{20} = \frac{0.05}{1+0.05(20)} = 0.025 = 2.50\%$$

62. You are given:

   a. Interest is earned using simple interest.
   b. $v(10) = 0.5$

   Determine δ_{20}.

   Solution:

$$v(t) = \frac{1}{a(t)} \to v(10) = 0.5 = \frac{1}{1+10s}$$

$$0.5 + 5s = 1$$

$$s = 0.1$$

$$\delta_{20} = \frac{a'(t)}{a(t)} = \frac{s}{1+st} = \frac{0.1}{1+0.1(20)} = 0.033333 = 3.333\%$$

63. You are given that $\delta_t = 0.01t$. John invests Z at time equal to zero. At time 5, he has 5000. Calculate Z.

   Solution:

$$Za(5) = 5000$$

$$Z = \frac{5000}{a(t)} = \frac{5000}{e^{\int_0^5 0.01t\,dt}} = \frac{5000}{e^{\left[\frac{0.01t^2}{2}\right]_0^5}} = \frac{5000}{e^{0.125}} = 4412.48$$

64. You are given $\delta_t = 0.04 + 0.003t^2$. Binbin is going to receive 9000 at the end of 10 years.

Calculate the value of this payment today.

Solution:

$$Xa(t) = 9000$$
$$a(t) = e^{\int_0^{10} 0.04+0.003t^2} = e^{\left[0.04t+\frac{0.003t^3}{3}\right]_0^{10}} = e^{1.4}$$
$$X = \frac{9000}{e^{1.4}} = 2219.37$$

65. You are given that $\delta_t = 0.05 + 0.006t$. Deepa invests 4000 at time 5.

Calculate the amount that Deepa will have at time 10.

Solution:

In this problem, we need to make sure that we pay attention to the time that the money was invested.

$$x = 4000a(t)$$

$$a(t) = e^{\int_5^{10} 0.05+0.006t\,dt} = e^{\left[0.05t+0.003t^2\right]_5^{10}} = e^{0.8-0.325} = e^{0.475}$$

$$4000e^{0.475} = 6432.06$$

66. Malcolm invests 2000 in an account today.

For the first two years, Malcolm earns an annual effective rate of discount of 5%.

During the next three years, Malcolm earns a nominal annual rate of interest of 12% compounded monthly.

Finally, during the last four years, Malcolm earns a force of interest of $\delta_t = 0.01t$ where t is measured from today.

Calculate that amount that Malcolm will have in the fund at the end of 9 years.

Solution:

$$= 2000(1-0.05)^{-2}\left(1+\frac{0.12}{12}\right)^{12(3)} e^{\int_5^9 0.01t\,dt}$$

$$= 2000(1-0.05)^{-2}\left(1+\frac{0.12}{12}\right)^{12(3)} e^{\left[0.005t^2\right]_5^9}$$

$$= 2000(1-0.05)^{-2}\left(1+\frac{0.12}{12}\right)^{12(3)} e^{0.28}$$

$$= 4195.22$$

67. A bank account pays interest at a force of interest of $\delta_t = 0.001t^2$.

Rachel invests X at time 15. At time 20 (five years after her investment), Rachel has 100,000.

Calculate X.

Solution:

$$xe^{\int_{15}^{20} 0.001t^2 dt} = 100000$$
$$xe^{\left[\frac{0.001}{3}t^3\right]_{15}^{20}} = 100000$$
$$xe^{1.54166667} = 100000$$
$$x = 21402.41$$

68. You are given:

a. $a(t) = 1 + ct + 0.001t^3$ where c is a constant

b. $\delta_{10} = 0.14$

Lexi invests 1000 at time 0. How much will she have at time 10?

Solution:

$$\delta_t = \frac{a'(t)}{a(t)}$$

$$a(t) = 1 + ct + 0.001t^3$$

$$a'(t) = c + 0.003t^2$$

Note: $\delta_{10} = \frac{a'(10)}{a(10)} = \frac{c + 0.3}{1 + 10c + 1} = 0.14 = \frac{c + 0.3}{2 + 10c}$

$$0.28 + 1.4c = c + 0.3$$

$$0.4c = 0.02$$

$$c = 0.05$$

$$1000a(10) = 1000\left(1 + 0.05(10) + 0.001(10^3)\right) = 2500$$

69. Cory can invest 1000 in either of the following accounts:

c. Account A earns compound interest at an annual effective rate of i
d. Account B earns interest equivalent to an annual effective discount rate of 6% for the first five years. Thereafter, Account B accumulates at a $\delta_t = 0.05 + 0.002t$.

At the end of 10 years Cory would have the same amount in either account.

Calculate i .

Solution:

Account A: $1000(1+i)^{10}$

Account B: $1000(1-0.06)^{-5} e^{\int_5^{10} 0.05+0.002t dt}$

At the end of 10 years, A ad B have the same amount.

$$1000(1+i)^{10} = 1000(1-0.06)^{-5} e^{\int_5^{10} 0.05+0.002t dt}$$

$$(1+i)^{10} = (1-0.06)^{-5} e^{\left[0.05t+0.001t^2\right]_5^{10}} = (1-0.06)^{-5} e^{0.325} = 1.8858$$

$$i = \sqrt[10]{1.8858} - 1 = 6.54931\%$$

Chapter 1, Section 1.14

70. Calculate the nominal interest rate if the real interest rate is 6% and the rate of inflation is 2%.

Solution:

(1+nominal interest rate)=(1+real interest rate)(1+inflation rate)

$$(1+i) = 1.06 * 1.02$$

$$i = 0.0812 = 8.12\%$$

71. Brad invests money in a bank account. During the year, his real rate of interest is 3% even though the rate of interest is on his account is 8%. Calculate the rate of inflation.

Solution:

$$(1+r) = \frac{1.08}{1.03}$$

$$r = 4.8544\%$$

72. Kristen invests in an account that pays annual rate of interest of 7.1%. The annual rate of inflation is 2%. Calculate Kristen's real rate of return.

Solution:

$$(1+j)=\frac{1.071}{1.02}$$
$$j=5\%$$

73. A gallon of gasoline costs 3.00 today. Lewis has enough money to buy 100 gallons today.

Instead of buying gasoline, Lewis decides to invest his money at an annual interest rate of 6.6%.

If the annual rate of inflation over the next five years is 4.1%, calculate how many gallons of gasoline will Lewis be able to buy at the end of five years.

Solution:

Today Lewis has 3(100)=$300

In 5 years Lewis will have $300(1.066)^5=\$412.9593258$

In 5 years a gallon will cost $3(1.041)^5=\$3.66754$

Lewis will be able to buy $\frac{412.9593258}{3.66754}=112.598$ gallons

74. Today, the cost of a song on iTunes is 1.00. Drew has 100 today and could use that 100 to buy 100 songs on iTunes. Instead, Drew decides to invest the 100 at an annual effective rate of interest of 8% for the next four years. At the end of four years, Drew can buy 117 songs on iTunes. Calculate the annual effective inflation rate on iTune songs over the four year period.

Solution:

$$100(1.08)^4=136.048896$$

$$\frac{136.048896}{New\Pr ice}=117 \text{ ==> } New\Pr ice=1.1628$$

$$Old\Pr ice(1+r)^4=New\Pr ice \text{ ==> } (1+r)^4=\frac{1.1628}{1.000}$$

$$r=3.843\%$$

Answers

1. 1,252,500
2. 68,887
3. 6.8619
4. 225.0256
5.
    a. 5000
    b. 5000
    c. 5500
    d. 1.1
    e. 500
    f. 10%
6.
    a. 3187.50
    b. 7.2165%
    c. 220
7. 1.15752
8. $\alpha = 1; \beta = 0.10; \delta = 0.05$
9.
    a. Investment 2 is better for 5 years as a(5) for Investment 2 is 1.25 while it is only 1.125 for Investment 1
    b. Investment 1 is better for 15 years as a(15) for Investment 1 is 4.375 while it is only 3.25 for Investment 2
    c. 10 years
10. $\frac{n(n+1)}{2}$
11. $2^{n+1} - 2$
12.
    a. 126
    b. 126
    c. 4320
    d. 14.2857 years
13. 6%
14. 5%
15. 12,300
16. 7.1429%
17. 30,000
18. 11
19.
    a. 126
    b. 324.90
    c. 6965.43
    d. 10.2448 years

20. 4.8809%
21. 1313.65
22. 11th year
23. 4.9942%
24. 1219.06
25. 167.82
26. 800 and 9200
27.
    a. 8.3333%
    b. 9.0909%
28. 6.5292%
29. 0.6410
30. 8064.52
31. 7903.15
32. 2.5
33. 10,097.08 and 3993.83 more
34. 500
35. 4.3489%
36. 10%
37. NPV=4.7870 and IRR=15.8014%
38. 15.913%
39.
    a. 19,518.72
    b. 5481.28
    c. 6.3830%
40. 16,324.49
41. 10,273.69
42. 248.69
43. $i = 5.31137\%; X = 2934.48$
44. 6.6967%
45. Tonia should take loan a because she has to repay the least at the end of 3 years. Under loan a, she has to pay 1300. Under loan b, Tonia will have to pay 1302.17. Under loan c, she would have to pay 1305.38.
46.
    a. 0.6667%
    b. 8.3000%
    c. 7.6639%
    d. 2.0134%
47.
    a. 0.7207%
    b. 8.6488%
    c. 8.4347%

48.
   a. 1.5000%
   b. 6.2319%
   c. 6.0607%
49. 6543.23
50. 3.6122%
51. 78,599.10
52. 12.25 years
53. 770.07
54. 796.86
55. You should pick Bank B. The annual effective interest rate for Bank A is 8.085% while the annual effective interest rate for Bank B is 8.000%. Since William is taking a loan, he wants the lowest interest rate.
56. 2028.05
57.
   a. 140,000.00
   b. 180,187.25
58. 9686.41
59. 8.9002%
60. 1,006,451.45
61. 319,819.02
62.
   a. $\dfrac{0.006t}{1+0.003t^2}$
   b. 4.6154 %
63. 7.3914%
64. 2.7273%
65. 3.33333%
66. 4412.48
67. 2219.37
68. 7459.12
69. 4195.22
70. 45,308.90
71. 2500
72. 6.5493%
73. 2.8846%
74. 5.8252%
75. 5.0000%
76. 112.675 gallons
77. 3.8430%