MATH 373
Chapter 1 Homework

Non-Interest Theory

1. 2005 + 2010 + … + 3500 =

2. 3 + 12 + 48 + . . . + 3,145,728=

3. $1 + 0.92 + 0.92^2 + . . . 0.92^{12} =$

4. If $(1+i)^5 = 1.15,$ calculate $1 + (1+i)^5 + (1+i)^{10} + ... + (1+i)^{100}$.

Section 1.3

5. Ayanna borrows 10,000 from Ben. After 4 years, Ayanna repays the loan by paying Stephen 13,000.
   a. Calculate the principle in this transaction?
   b. How much interest does Ayanna pay?
   c. Determine $A(0)$.
   d. Determine $A(4)$.
   e. Determine $a(4)$.

6. Maggie lends 5000 to Liam who repays the loan after 3 years. You are given that $a(t) = 1 + 0.04t + 0.001t^3$. Calculate the amount that Liam will have to repay at the end of 3 years.

7. You are given that $a(t) = \alpha + \beta t^2$. You are also given that 1000 invested today will grow to 1360 at the end of 3 years.

   Determine α and β.

8. Yujie invests 4000 in a fund that has an accumulation function of $\alpha + \beta t + \omega t^2$. At the end of two years, Yujie has 4200. At the end of four years, Xinyi has 4560.

   Determine the amount that Yujie has at the end of eight years.

MATH 373
Chapter 1 Homework

9. You are given that $a(t) = 1 + 0.04t + 0.001t^3$.

   Calculate:
   a. $i_{[2,4]}$
   b. The amount of interest that Clay will earn from time 2 until time 4 assuming he invests 1000 now
   c. $i_{[3,4]}$
   d. i_4
   e. The annual effective interest rate during the fourth year.
   f. $i_{[2.5,3.2]}$

10. You are given that:

    a. $i_1 = 0.04$
    b. $i_2 = 0.055$
    c. $a(3) = 1.184976$

    Calculate i_3.

11. Alan has a choice of two investments. Investment 1 has an accumulation function of $a(t) = 1 + 0.001t^3$. Investment 2 has an accumulation function of $a(t) = 1 + 0.01t^2$.

    a. Which investment should Alan use if he will be investing for 5 years? Explain why.
    b. Which investment should Alan use if he will be investing for 15 years? Explain why.
    c. If Alan invests 1000 into both investments, at what time will the accumulated value of both investments be equal?

Section 1.4

12. Cady invests 1250 in a fund earning simple interest at a rate of 4.6% per year.

    a. Write an expression for $a(t)$.
    b. Write an expression for $A(t)$.
    c. Determine the amount of interest that Cady will earn in the first year.
    d. Determine the amount of interest that Cady will earn in the 9th year.
    e. How much money will Cady have after 10 years?
    f. Calculate i_5 and i_{10}.

13. Ayanna borrows 10,000 from Ben. After 4 years, Ayanna repays the loan by paying Ben 13,000. If Ayanna is paying simple interest, determine the simple rate of interest.

14. Ty borrows K to buy a new television set. Ty pays simple interest of 12% on the loan. Ty repays the loan at the end of 9 months with a payment of 654.

    Determine K.

15. Cora invests 1000 in a fund earning simple interest at a rate of $s\%$.

    Cora earns an effective rate of interest of 6.25% during the 9th year of the investment.

    Determine the amount the Cora will have after 10 years.

16. Jolly invests in a fund earning simple interest of 5%.

    The effective interest rate for the nth year (i_n) is 2.5%.

    Calculate n.

Section 1.5

17. Cady invests 1250 in a fund earning compound interest at a rate of 4.6% per year.

    a. Write an expression for $a(t)$.
    b. Write an expression for $A(t)$.
    c. Determine the amount of interest that Cady will earn in the first year.
    d. Determine the amount of interest that Cady will earn in the 9th year.
    e. How much money will Cady have after 10 years?
    f. Calculate i_5 and i_{10}.

18. Jin invests 10,000 in an account earning compound interest at a rate of i. After 6 years, Jin has 14,387.11.

    Determine i.

19. Kaitlyn borrows 2000 at a compound interest rate of 5%. She repays the loan at the end of n years with a payment of 4584.04.

    Determine n.

MATH 373
Chapter 1 Homework

20. Xin invests 1300 in a fund that pays compound interest for 9 years.

    The interest rates are:
    i.   5% for the first 4 years;
    ii. 4% for the next 3 years; and
    iii. 6.5% for the last 2 years.

    Determine the amount Xin will have after 9 years.

    Determine the level annual effective interest rate that is equivalent to the rates that Xin earned.

21. Ian invests money in a bank account earning compound interest at an annual effective interest rate of 5%.

    Hannah invests money in a bank account earning 10% simple interest.

    What year will Ian and Hannah earn the same annual effective interest rate?

22. Ryan invests 25,000 in an account earning simple interest of 5%.

    Chelsea invests X in an account earning 2% compounded annually.

    In year Y, Chelsea and Ryan earn the same annual effective interest rate.

    At the end of year Y, the amount of money in Chelsea's account is equal to the amount of money in Ryan's account.

    Determine X.

23. Allison invests 100 in an account earning simple interest. During the 10th year, the amount of interest that Allison earns is 20.

    Elijah invests X into an account earning compound interest of i. During the 10th year, the amount of interest that Elijah earns is also 20.

    During the 11th year, Allison and Elijah earn the same annual effective interest rate.

    Determine X .

MATH 373
Chapter 1 Homework

Section 1.6

24. Jia borrows 1600 from a bank to buy a new refrigerator. The bank charges interest in advance at a rate of 9%. Jia repays the loan after one year.

    a. Determine the principle for Jia's loan.

    b. Determine the amount of discount for this loan.

    c. Determine the amount of money that Jia had to spend on a computer at time 0.

    d. Determine the amount of money that Jia will need to pay at time 1 to repay the loan.

25. You are given that $a(t) = (1.08)^t$.

    Calculate:

    a. $d_{[3,4]}$

    b. d_4

    c. d_{10}

    d. $d_{[2.5,4]}$

26. You are given that $a(t) = 1 + 0.08t$.

    Calculate:

    a. $d_{[3,4]}$

    b. d_4

    c. d_{10}

27. If $i_5 = 0.05$, calculate d_5.

28. If $d_5 = 0.05$, calculate i_5.

MATH 373
Chapter 1 Homework

Section 1.7

29. You are given that $a(t) = 1 + 0.04t + 0.001t^3$.

    Calculate:
    a. $v(5)$.
    b. The amount that you must invest today to have 10,000 at the end of 5 years.

30. Edwin borrows money to buy a car. The loan has a simple interest rate of 8%. At the end of 4 years, Edwin repays the loan with a payment of 13,000.

    Calculate the amount of the loan.

31. Edwin borrows money to buy a car. The loan has compound interest at a rate of 8%. At the end of 4 years, Edwin repays the loan with a payment of 13,000.

    Calculate the amount of the loan.

32. Lucy wants to have 5000 at the end of 5 years. If Lucy invests in a fund earning simple interest of 7%, determine the amount that Lucy must invest today to have 5000 at the end of 5 years.

33. Lucy wants to have 5000 at the end of 10 years. Lucy invests in a fund earning simple interest of 7%. Lucy invests D at the end of 5 years and it grows to 5000 at the end of 10 years.

    Determine D.

34. You are given that $v(t) = \dfrac{1}{(\alpha t + \beta)}$.

    Using this discount function, a payment of 200 to be made at time 10 has a present value of 100 at time 0.

    Calculate $a(20)$.

35. The present value of 12,345 at the end of 7 years is 8,765 assuming a compound annual interest rate of i.

    Calculate i.

MATH 373
Chapter 1 Homework

36. Zhang Corporation is building a new factory. Zhang expects the factory will generate the following cash flows:

Time	Cash Flow
0	- 700,000
1	- 100,000
2	300,000
3	400,000
4	300,000
5	200,000

   a. Calculate the Net Present Value at an annual effective interest rate of 10%.
   b. Calculate the Net Present Value at an annual effective interest rate of 20%.
   c. Calculate the Internal Rate of Return (IRR).
   d. What is the Net Present Value at the IRR. (Note: You should be able to answer this question without doing any work.)

37. Li Corporation invests X million today to build a factory. The factory is expected to produce the following profits:

End of Year	Profits
1	1 million
2	4 million
3	2 million
4	1 million

At the end of 4 years, the factory will be obsolete and will be closed.

The Net Present Value of this project to Fisher Corporation is 1 million at an interest rate of 8%.

Calculate the Internal Rate of Return on this project.

MATH 373
Chapter 1 Homework

38. Knable Incorporated manufactures farm equipment. Knable is going to build a new factory. Based on projections, it is expected that Knable will need to invest 5,700,000 at the beginning of this project. Additional cash flows over the next five years will be as follows:

End of Year	Cash Flow
1	-1,000,000
2	X
3	2,000,000
4	4,000,000
5	5,000,000

   After five years, the factory will be obsolete and not generate any additional cash flows.

   This factory based on expected cash flows will generate an internal rate of return of 10%.

   Calculate the Net Present Value of the expected cash flows at an annual effective rate of 11%.

Section 1.9

39. Mengzhi borrows 18,000. The loan is repaid in 5 years at an annual effective rate of discount of 8.25%.

   Calculate:
   a. The amount of cash that Mengzhi will receive at time 0.
   b. The amount of discount that Mengzhi will pay.
   c. The equivalent compound annual interest rate.

40. Kelly wants to buy a new car. The bank offers her a loan which has a compound annual discount rate of 6.5%. Kelly needs 12,500 to buy the car and want to repay the loan at the end of 4 years.

   How much must Kelly borrow (repay in 4 years) to be able to have 12,500 now to buy the car?

41. Rachel needs 20,000 to pay her tuition for this year. She has the option of the following two loans:
   a. A three year loan at a compound annual interest rate of 7%;
   b. A three year loan at a compound annual discount rate of 6.55%.

   Which loan should Rachel accept and why?

42. Keith has a loan with a bank where the terms call for him to receive 10,000 today and to repay 14,500 at the end of five years. Determine the annual compound discount rate on this loan.

MATH 373
Chapter 1 Homework

43. Fatin invests 23,000 in an account that pays interest at a rate equivalent to a 5.75% discount rate. Determine the amount of money that Fatin will have at the end of 6 years.

44. Matt invests 100 in a bank account earning a simple interest rate of 10%. Kasey invests 100 in a bank account which earns an annual effective discount rate of d. At the end of 10 years, Matt and Kasey have the same amount of money in their accounts. Calculate d.

Section 1.10

45. Jessica borrows 3000. Jessica's loan will charge interest at a rate of 6% compounded quarterly. The loan is repaid at the end of 30 months.

    Calculate the amount that Jessica must pay to repay the loan.

46. Tong borrows money to buy a new car. The loan has an interest rate of 9% compounded monthly. At the end of 4 years, Tong repays the loan with a payment of 36,000.

    Calculate the amount that Tong borrowed.

47. Nanqi invests 1000 in an account earning a nominal interest rate which is compounded quarterly. At the end of 6 years, Nanqi has 2000.

    Determine the nominal interest rate compounded quarterly that Nanqi is earning.

48. You are given that $i^{(12)} = 0.12$. Calculate the equivalent:
    a. Monthly effective interest rate
    b. Annual effective interest rate
    c. $i^{(4)}$
    d. $d^{(2)}$

49. Colin borrows 10,000 at an interest rate of 5% compounded monthly. At the end of the loan, Colin pays 15,474.23.

    Calculate the length of Colin's loan in years.

MATH 373
Chapter 1 Homework

50. Patric invest 40,000 in a fund for nine years. The fund earns the following:

    i.   For the first four years, a discount rate of 9% convertible monthly;

    ii. For the next three years, an interest rate of 6% compounded quarterly;

    iii. For the last two years, a quarterly effective interest rate of 4%.

    Determine the amount of money that Patric will have at the end of nine years.

51. Emily borrows 20,000 for 8 years at a nominal discount rate of 6% compounded quarterly. Calculate the amount of discount that Emily will pay.

52. Chufan invests 13,000 in an account that earns interest at a rate equivalent to a discount rate of 6% convertible semi-annually. How much will Chufan have after 18 months.

53. Chad wants to borrow money to buy a car.

    Bank A offers Chad a loan with an annual effective interest rate of 8.0%.

    Bank B offers Chad a loan with an interest rate of 7.8% compounded monthly.

    Which loan should Chad select and state why. (Provide work to support your answer.)

54. DeVonna invests 100,000 in an account that earns a nominal interest rate of 10% compounded every 4 years.

    a. Calculate the amount that DeVonna will have at the end of 4 years.
    b. Calculate the amount that DeVonna will have at the end of 7 years.

55. Carl invests 10,000 in an account earning a nominal interest rate of 9% convertible every two years.

    Brian invests K in an account earning a nominal rate of interest of 9% convertible monthly.

    After 10 years, the amount in Carl's account is equal to the amount in Brian's account.

    Determine K.

Section 1.11

56. You are given that $i^{(12)} = 0.07$. Calculate δ.

57. Taufiq invests 17,500 in an account that earns 8.25% interest compounded continuously. Determine the amount that Taufiq will have after 6 years.

58. Zixin borrows 1800 today. Zixin must repay the loan using a continuously compounded interest rate of δ. He repays 3500 at the end of 10 years.

   Determine δ.

Section 1.12

59. You are given that $a(t) = 1 + 0.03t^2$. Calculate:

   a. δ_1
   b. δ_5

60. You are given:

$$v(t) = \frac{1}{(1+0.02t)(1+0.04t^2)}$$

   Calculate δ_6.

61. Under simple interest with an interest rate of 5%, calculate δ_{20}.

62. You are given:

   a. Interest is earned using simple interest.
   b. $v(10) = 0.4$

   Determine δ_{20}.

MATH 373
Chapter 1 Homework

63. You are given that $\delta_t = 0.02t$.

    a. Connor invests 4000 today. Determine the amount that Connor will have at the end of 3 years.

    b. Grant invests 4000 at the end of 7 years. Determine the amount that Grant will have at the end of 10 years.

64. Samuel wants to have 50,000 to make a down payment on a house at the end of 10 years. Samuel deposits D into an account today that will pay a force of interest of $0.01t + 0.003t^2$.

    Determine the minimum D so that Samuel meets his goal.

65. Samantha invests 1000 in an account earning $\delta_t = 0.02t$. At the end of t years, Kehara's investment has doubled.

    Calculate t (t will not be an integer.)

66. You are given:

    a. $a(t) = 1 + ct + 0.001t^3$ where c is a constant
    b. $\delta_{10} = 0.14$

    Miao invests 1000 at time 0. How much will Miao have at time 10?

MATH 373
Chapter 1 Homework

67. Collin can invest 1000 in either of the following accounts:

    a. Account A earns compound interest at an annual effective rate of i

    b. Account B earns interest equivalent to an annual effective discount rate of 6% for the first five years. Thereafter, Account B accumulates at $\delta_t = 0.05 + 0.002t$.

    At the end of 10 years Collin would have the same amount in either account.

    Calculate i.

Section 1.13

68. You are given that the nominal interest is 8% and the rate of inflation is 2.5%.

    Calculate the inflation adjusted interest rate.

69. If the real interest rate is 6% and the rate of inflation is 3%, calculate the nominal interest rate.

70. Chang has 100 and could use it to buy 80 songs from itunes. Instead, Chang invests his 100 at an annual effective interest rate of 11.3% for 4 years. The price of songs on itunes is subject to an annual effective rate of inflation of $r\%$. At the end of 4 years, Chang can buy 101 songs.

    Determine r.

71. A gallon of gasoline costs 3.00 today. Shannon has enough money to buy 100 gallons today.

    Instead of buying gasoline, Shannon decides to invest his money at an annual interest rate of 6.6%.

    If the annual rate of inflation over the next five years is 4.1%, calculate how many gallons of gasoline Shannon will be able to buy at the end of five years.

MATH 373
Chapter 1 Homework

Answers

1. 825,750
2. 4,194,303
3. 8.27184
4. 118.81012
5.
    a. 10,000
    b. 3000
    c. 10,000
    d. 13,000
    e. 1.30
6. 5735
7. $\alpha = 1$ and $\beta = 0.04$
8. 5760
9.
    a. 12.5%
    b. 136
    c. 6.7132%
    d. 6.7132%
    e. 6.7132%
    f. 4.0464%
10. 8.00%
11.
    a. Investment 2 is better for 5 years as a(5) for Investment 2 is 1.25 while it is only 1.125 for Investment 1
    b. Investment 1 is better for 15 years as a(15) for Investment 1 is 4.375 while it is only 3.25 for Investment 2
    c. 10 years
12.
    a. $a(t) = 1 + 0.046t$
    b. $A(t) = 1250[1 + 0.046t]$
    c. 57.50
    d. 57.50
    e. 1825
    f. 3.8851% and 3.2532%
13. 7.50%
14. 600
15. 2250
16. 21

17.
   a. $a(t) = (1.046)^t$

   b. $A(t) = 1250(1.046)^t$

   c. 57.50
   d. 82.40
   e. 1959.87
   f. 4.6% and 4.6%
18. 6.25%
19. 17
20. 2016.04 and 4.9960%
21. 11
22. 34,504.43
23. 167.83
24.
   a. 1600
   b. 144
   c. 1456
   d. 1600
25.
   a. 7.4074%
   b. 7.4074%
   c. 7.4074%
   d. 10.9027%
26.
   a. 6.0606%
   b. 6.0606%
   c. 4.4444%
27. 4.7619%
28. 5.2632%
29.
   a. 0.754717
   b. 7547.17
30. 9848.48
31. 9555.39
32. 3703.70
33. 3970.59
34. 3
35. 5.0143%
36.
   a. 86,639
   b. -118,467
   c. 13.74662%
   d. 0

37. 15.9131%
38. -253,439.23
39.
     a. 11,703.15
     b. 6296.85
     c. 8.9918%
40. 16,355.49
41. Rachel should select loan a. The annual effective interest rate on loan a. is 7% while the annual effective rate on loan b. is 7.0091%. Since Rachel is taking a loan, she would want the lower annual effective interest rate.
42. 7.1619%
43. 32,812.53
44. 6.6967%
45. 3481.62
46. 25,150.11
47. 11.7209%
48.
     a. 1.0000%
     b. 12.6825%
     c. 12.1204%
     d. 11.5910%
49. 8.75 years
50. 93,940.86
51. 7669.25
52. 14,243.87
53. Chad should choose Bank A because the annual effective interest rate is lower.
54.
     a. 140,000
     b. 180,187.25
55. 9332.62
56. 6.9797%
57. 28,708.72
58. 6.6498%
59.
     a. 5.8252%
     b. 17.1429%
60. 21.4578%
61. 2.5%
62. 3.75%
63.
     a. 4376.70
     b. 6661.16
64. 11,156.51

65. 8.3255 years
66. 2500
67. 6.5493%
68. 5.3659%
69. 9.18%
70. 5%
71. 112.598 gallons