1. The Purdue Life Insurance Company has two assets and two liabilities.

The assets are:
   a. A 5 year par value bond with a maturity value of 100,000. The bond pays annual coupons at a rate of 6%; and
   b. A zero coupon bond which matures in 10 years for 250,000.

The liabilities are:
   c. An annuity immediate with payments of 35,000 at the end of each year for 5 years; and
   d. An perpetuity immediate with annual payments of 5,000.

Calculate the owners equity (surplus) of the Purdue Life Insurance Company assuming an interest rate of 8%.

Calculate the owners equity (surplus) of the Purdue Life Insurance Company assuming an interest rate of 12%.

2. John must pay Kristen 10,000 at the end of 1 year. He also must pay Ahmad 30,000 at the end of year 2.

John wants to exactly match his liabilities by purchasing the following two bonds:
   a. Bond A is a one year zero coupon bond maturing for 1000.
   b. Bond B is a two year bond with annual coupons of 200 and a maturity value of 1000.

Calculate the amount of each bond that John should buy.

3. Trena has agreed to pay Jayme the following payments:
   a. 30,000 at the end of 1 years;
   b. 50,000 at the end of 2 years; and
   c. 10,000 at the end of 3 years.

You are given the following three bonds:
   a. A one year bond with annual coupons of 100 and a maturity value of 2000 with a price of 2000.
   b. A two year bond with annual coupons of 80 and a maturity value of 1000 with a price of 1020.
   c. A three year bond with annual coupons of 50 and a maturity value of 1800 with a price of 1580.

Determine the amount of the two year bond that should be purchased to exactly match the cash flows of Trena’s liability. (Assume that you can purchase partial bonds.)
4. Rivera Insurance Company has committed to paying 10,000 at the end of one year and 40,000 at the end of two years. It’s Chief Financial Officer, Miguel, wants to exactly match this obligation using the following two bonds:

   Bond A is a one year bond which matures at par of 1000 and pays an annual dividend at a rate of 6%. This bond can be bought to yield 6% annually.

   Bond B is a two year bond which matures at par of 1000 and pays an annual dividend at a rate of 10%. This bond can be bought to yield 7% annually.

   Calculate the amount of each bond that Rivera should purchase.

   Calculate the cost of Rivera to exactly match this obligation.

5. Wang Life Insurance Company issues a three year annuity that pays 40,000 at the end of each year. Wang uses the following three bonds to absolutely match the cash flows under this annuity:

   a. A zero coupon bond which matures in one year for 1000.
   b. A two year bond which matures for 1200 and pays an annual coupon of 100. This bond is priced using an annual yield of 7%.
   c. A three year bond which matures for 2000 and pays annual coupons of 75. This bond has a price of 1,750.

   It cost Wang 104,000 to purchase all three bonds to absolutely match this annuity.

   Calculate the one year spot interest rate.

6. Ace is receiving an annuity immediate with level annual payments of 500 for 18 years.

   Calculate the Macaulay duration and the Modified duration at an annual effective interest rate of 6%.

7. Li Life Insurance Company is paying Cora an annuity due of 234 per year for the next 10 years.

   Calculate the Modified duration of Cora’s annuity at an annual effective interest rate of 10%.

8. A 20 year bond has annual coupons of 400. The bond matures for 13,000.

   Calculate the Macaulay Duration of this bond at an annual effective interest rate of 5.5%.

9. Mayfawny owns an 8 year bond with a par value of 1000. The bond matures for par and pays semi-annual coupons at a rate of 6% convertible semi-annually.

   Calculate the Modified duration of this bond at an annual effective interest rate of 8.16%.
10. A five year bond matures for 20,000. The bond pays coupons of:
   a. 3000 at the end of the first year,
   b. 1500 at the end of the second year,
   c. 1000 at the end of the third year,
   d. 750 at the end of the fourth year, and
   e. 600 at the end of the fifth year.

Calculate the Macaulay Duration of this bond at 5%.

11. James has a loan of 10,000 which is to be repaid with 10 level annual payments at an annual effective interest rate of 12%. Calculate the Macaulay duration of the loan using the 12% interest rate.

12. Trena has agreed to pay Jayme the following payments:
   a. 30,000 at the end of 2 years;
   b. 50,000 at the end of 5 years; and
   c. 10,000 at the end of 8 years.

You are given that \( v = 0.9 \).

Calculate the Modified Duration of Trena’s liability.

13. Ivey Investments owns a preferred stock which pays a quarterly dividend of $5 per quarter with the next dividend paid in 3 months. Calculate the modified duration of this stock at an annual effective rate of 8.243216%.

14. The Macaulay Duration of a perpetuity immediate with level annual payments is 26.

Determine the interest rate that was used to calculate the Macaulay Duration.

15. Sparks-Norris Asset Partners (SNAP) manages the following portfolio of bonds:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Price</th>
<th>Macaulay Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5,000</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10,000</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>20,000</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>15,000</td>
<td>2</td>
</tr>
</tbody>
</table>

The duration is calculated at an annual effective interest rate of 7%.

Calculate the modified duration of SNAP’s portfolio.
16. An annuity immediate pays 100 at the end of each year for 5 years. Calculate the Macaulay convexity and the Modified convexity of this annuity at an annual effective rate of 6%.

17. Calculate the modified convexity for a 3 year bond with annual coupons of 300 and a maturity value of 5000 using an annual effective interest rate of 8%.

18. A 3 year bond has annual coupons.

   The coupon at the end of the first year is 100.

   The coupon at the end of the second year is 300.

   The coupon at the end of the third year is 500.

   The bond matures for 700.

   Calculate the modified convexity of this bond at an annual effective rate of interest of 6%.

19. John is receiving an annuity due with three annual payments. The first payment is 100, the second payment is 200, and the last payment is 300.

   John calculates the Macaulay Covexity of his annuity to be 2.20268 at an annual effective interest rate of \( i \).

   Determine \( i \), the interest rate used by John. (Hint: Find \( v \) and then find \( i \).)

20. A bond has a Macaulay Duration of 4.450926 and a Macaulay Convexity of 21.19773 when calculated using an annual effective interest rate of 8%. The price of the bond is 1096.19.

   A. Estimate the price of the bond if the annual interest rate increases to 8.5% using only the duration.

   B. Estimate the price of the bond if the annual interest rate increases to 8.5% using both the duration and the convexity.

   C. The values in this problem are based on a 5 year bond with annual coupons of 70 and a maturity value of 1200. What would the actual price be at 8.5%.
21. The Wang Insurance Company has the following two assets:
   
a. A zero coupon bond which matures for 10,000 in five years and has an annual yield of 8%.
   
b. A bond with a price of 20,000 which has a Macaulay Duration of 3 and a Macaulay Convexity of 11. All values were calculated at an annual effective interest rate of 8%.
   
Let $D_{Port}^{Mod}$ be the Modified Duration of this portfolio of assets at an interest rate of 8%.

Let $C_{Port}^{Mod}$ be the Modified Convexity of this portfolio of assets at an interest rate of 8%.

Calculate $C_{Port}^{Mod} - D_{Port}^{Mod}$.

22. Jenna owns the following portfolio.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Price</th>
<th>Macaulay Duration</th>
<th>Macaulay Convexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond 1</td>
<td>25,000</td>
<td>6.0</td>
<td>40</td>
</tr>
<tr>
<td>Bond 2</td>
<td>30,000</td>
<td>4.5</td>
<td>25</td>
</tr>
<tr>
<td>Bond 3</td>
<td>45,000</td>
<td>3.0</td>
<td>12</td>
</tr>
</tbody>
</table>

The price, Macaulay Duration, and Macaulay Convexity were calculated at an annual effective rate of 5%.

Estimate the price of the portfolio at an annual effective rate of interest of 7% using both the duration and convexity.

23. Sue wants to fully immunize a future payment of 100,000 at time 10 using the following two bonds:
   
a. A zero coupon bond maturing in 5 years; and
   
b. A zero coupon bond maturing in 20 years.
   
Determine the amount that Sue should spend on each bond at an annual effective interest rate of 10%.

24. Lauren wants to fully immunize a future payment of X at time Y using the following two bonds:
   
a. Bond A is a zero coupon bond maturing in 2 years; and
   
b. Bond B is a zero coupon bond maturing in 10 years.
   
Lauren pays 13,622.79 for Bond A and 6,192.18 for Bond B. Determine X and Y if the annual effective interest rate of 5%.
Answers

1.  5568 at 8% and – 8969 at 12%
2.  A => 5      B => 25
3.  46.05 of Bond B
4.  A => 6.0034305   B=> 36.363636
    Cost => 44,339.45
5.  5.2335%
6.  Macaulay => 7.9597   Modified => 7.5092
7.  3.3868
8.  14.179
9.  5.8732
10. 4.1824
11. 4.58
12. 3.57125
13. 11.78
14. 4%
15. 5.607
16.  Macaulay = 10.306543 and Modified = 11.7392
17. 9.507
18. 9.025
19. 11.111%
20.  A. 1073.60   B. 1073.90   C. 1073.90
21. 12.2376
22. 92.491.61
23. 25,702.89 on five year zero and 12,851.44 on twenty year zero.
24.  X = 24,680 and Y = 4.5