Mathematics 174

Division 2, Section 1

Test 2

March 28, 1988

SHOW YOUR WORK

(1) Calculate the gradient of the function

$$f(x, y) = \frac{e^x - e^y}{e^x + e^y}.$$

If $\mathbf{u} = \frac{1}{\sqrt{2}}(1, 1)$, then what is $f_{\mathbf{u}}(2, 1)$?

(2) Find the rate of change of the function $f(x, y) = x^2 + y^2$ along the curve $\mathbf{r}(t) = e^t \cos t\, \mathbf{i} + e^t \sin t\, \mathbf{j}$ with respect to t.

(3) Show that the curve $\mathbf{r}(t) = \frac{3}{2}(t^2 + 1)\, \mathbf{i} + (t^4 + 1)\, \mathbf{j} + t^3\, \mathbf{k}$ is perpendicular to the ellipsoid $x^2 + 2y^2 + 3z^2 = 20$ at the point $(3, 2, 1)$.

(4) Maximize the function $f(x, y) = xy$ on the ellipse $b^2x^2 + a^2y^2 = a^2b^2$.

(5) Is the vector function $(1 + e^y)\, \mathbf{i} + (xe^y + y^2)\, \mathbf{j}$ the gradient of a function $f(x, y)$ defined for all x and y? If so, find the most general function f with this gradient.

(6) Sketch the domain of integration Ω for the repeated integral

$$\int_0^1 \int_y^1 e^{y/x}\, dx\, dy.$$

Change the order of integration, and then evaluate the integral.