MA 174: Multivariable Calculus

EXAM II

Mar. 23, 2007

NAME _____

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) _____ **6. (5 pts)** _____

2. (5 pts) _____ **7. (5 pts)** _____

3. (5 pts) _____ **8. (5 pts)** _____

4. (5 pts) _____ **9. (5 pts)** _____

5. (5 pts) _____

Total Points: _____/45

1. If $xz + y\ln x - x^2 + 4 = 0$ defines x as a function of two independent variable y and z, find $\dfrac{\partial x}{\partial z}$ at $(x, y, z) = (1, -1, -3)$.

A. 0

B. $\boxed{\frac{1}{6}}$

C. $\frac{1}{5}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

2. In what direction is the derivative of $z = x^2 + 3xy - \frac{1}{2}y^2$ at $(-1, -1)$ equal to zero.

A. $3i$

B. $5\vec{i} + 2\vec{j} - \vec{k}$

C. $\boxed{2\vec{i} - 5\vec{j}}$

D. $2\vec{i} + 5\vec{j}$

E. $\sqrt{29}$

3. Find an equation for the tangent plane of $z = x^2 + 3xy - \frac{1}{2}y^2$ at $(-1, -1)$.

A. $5x + 2y + z - \frac{7}{2} = 0$

B. $5x + 2y - z - \frac{7}{2} = 0$

C. $5x - 2y + z - \frac{7}{2} = 0$

D. $5x - 2y - z + \frac{7}{2} = 0$

E. $\boxed{5x + 2y + z + \frac{7}{2} = 0}$

4. By using a linear approximation of $f(x,y) = x^2 + 3xy - \frac{1}{2}y^2$ at $(-1,-1)$, compute the approximate value of $f(-2,2)$.

A. 1.5

B. $\boxed{2.5}$

C. 3.5

D. 4.5

E. 5.5

5. Find the critical point of

$$f(x,y) = x^2 + 2y^2 - x$$

A. $(0,0)$

B. $(-1,1)$

C. $(\frac{1}{2},1)$

D. $\boxed{(\frac{1}{2},0)}$

E. $x = 1$

6. Find $\left(\dfrac{\partial w}{\partial x}\right)_y - \left(\dfrac{\partial w}{\partial x}\right)_z$ at $(w,x,y,z) = (6,1,1,2)$ if

$$w = x^2 + y^2 + z^2, \quad z = x^2 + y^2$$

A. -10

B. -5

C. 0

D. 5

E. $\boxed{10}$

7. Find the average value of $f(x,y) = y\cos(xy)$ over the rectangle $0 \le x \le 1$, $0 \le y \le \pi$.

A. 0

B. π

C. $\boxed{\frac{2}{\pi}}$

D. 2π

E. 2

8. (SHOW YOUR WORK) The extreme values of $f(x,y,z) = x - y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$ are _____, _____.

Answer: $\sqrt{3}$, $-\sqrt{3}$

9. (SHOW YOUR WORK) If $f(u,v,w)$ is differentiable, and $u = x - y$, $v = y - z$, $w = z - x$, write $\frac{\partial f}{\partial x}$ in terms of $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial v}$ and $\frac{\partial f}{\partial w}$,

$$\frac{\partial f}{\partial x} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

and then compute

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = \underline{\qquad\qquad}.$$

Answer: $\frac{\partial f}{\partial u} - \frac{\partial f}{\partial w}$, 0