MA 271: Several Variable Calculus

EXAM I

Sep. 25, 2007

NAME ____________________ Class Meet Time ____________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (10 pts) ________	**11. (5 pts)** ________
2. (5 pts) ________	**12. (5 pts)** ________
3. (5 pts) ________	**13. (5 pts)** ________
4. (5 pts) ________	**14. (5 pts)** ________
5. (5 pts) ________	**15. (5 pts)** ________
6. (5 pts) ________	**16. (5 pts)** ________
7. (5 pts) ________	**17. (5 pts)** ________
8. (5 pts) ________	**18. (5 pts)** ________
9. (5 pts) ________	**19. (5 pts)** ________
10. (5 pts) ________	

Total Points: ________/100

1. Determine convergence or divergence for the given sequences or series. Fill in the blanks.

(a) $\lim_{n\to\infty} \frac{1}{n}$ ____________________ **(converge, diverge)**

(b) $\lim_{n\to\infty} \sqrt[n]{2}$ ____________________ **(converge, diverge)**

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$ ____________________ **(converge, diverge)**

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ ____________________ **(converge, diverge)**

(e) $\sum_{n=1}^{100} \frac{1}{n}$ ____________________ **(converge, diverge)**

(f) $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{100}}$ ____________________ **(converge, diverge)**

(g) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ ____________________ **(converge, diverge)**

(h) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ ____________________ **(converge, diverge)**

(i) $\sum_{n=1}^{\infty} \sin(n\pi)$ ____________________ **(converge, diverge)**

(j) $\sum_{n=1}^{\infty} \frac{n^5}{e^n}$ ____________________ **(converge, diverge)**

2. True or False (meaning not always true or the formula does not make sense). For three-dimensional vectors a, b and c

(i) $\mathbf{b} = \mathbf{c}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ ____________________ (T, F)

(ii) $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$ ____________________ (T, F)

(iii) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ ____________________ (T, F)

(iv) $\mathbf{a} \times \mathbf{a} = |\mathbf{a}|^2$ ____________________ (T, F)

(v) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$ ____________________ (T, F)

3. Which of the following statements hold.

(i) $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(ii) $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges conditionally.

(iii) $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges.

(iv) $\sum_{n=1}^{\infty} a_n$ converges absolutely, then $\sum_{n=1}^{\infty} a_n$ converges conditionally.

(v) $\sum_{n=1}^{\infty} a_n$ converges conditionally, then $\sum_{n=1}^{\infty} a_n$ converges.

A. (i) and (ii)

B. (iii) and (iv)

C. (ii) and (iv)

D. (i) and (v)

E. (iii) and (v)

4. Evaluate the limit: $\lim_{n\to\infty} \sqrt[n]{2n^2} =$

A. 0

B. 1

C. $\sqrt{2}$

D. 2

E. diverge to ∞

5. Evaluate the limit:

$$\lim_{n\to\infty} (1 + \frac{5}{n})^{3n} =$$

A. e

B. e^3

C. e^{15}

D. 5^3

E. diverge to ∞

6. Evaluate the sum:

$$\sum_{n=-2}^{\infty} \frac{1}{3^n 2^n} =$$

(Note: the series starts from $n = -2$**.)**

A. $\frac{87}{2}$

B. $\frac{89}{3}$

C. $\frac{216}{5}$

D. $\frac{236}{5}$

E. diverge to ∞

7. Let x be a nonzero real number, what is the value of m if

$$\sum_{n=1}^{\infty} x^{n-6} = \sum_{n=m}^{\infty} x^n$$

- **A.** -5
- **B.** 0
- **C.** 1
- **D.** 5
- **E.** 6

8. What is the curvature of a straight line?

- **A. depend on the location of the line**
- **B. not defined**
- **C.** ∞
- **D.** 1
- **E.** 0

9. The intersection of the surface $y + 4 = (x-2)^2 + (z+2)^2$ and the xz-plane is

- **A. a straight line.**
- **B. two straight lines.**
- **C. a circle.**
- **D. a parabola.**
- **E. a hyperbola.**

10. Find the area of the parallelogram spanned by $\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\,\mathbf{j} - 3\,\mathbf{k}$.

A. 6

B. $\sqrt{11}$

C. $\frac{13}{2}$

D. $\sqrt{13}$

E. $\sqrt{14}$

11. Find the point of the intersection of two curves defined by

$$\textbf{curve 1:} \quad x = t^2, \quad y = t^2 - 1, \quad z = 1$$
$$\textbf{curve 2:} \quad x = 1 + s, \quad y = 4 - 3s, \quad z = 1$$

A. $(x, y, z) = (2, 3, 1)$

B. $(x, y, z) = (2, 1, 1)$

C. $(x, y, z) = (1, 2, 1)$

D. $(x, y, z) = (-3, -2, 0)$

E. **no intersection point**

12. Let L be the tangent line to the curve given by

$$x = 2\cos(t) + \sin(2t)$$
$$y = 2\sin(t) + \cos(2t)$$
$$z = 3t$$

at the point $(2, 1, 0)$. Which of the following point is on the tangent line L?

A. $(4, 2, 6)$

B. $(4, 3, 3)$

C. $(4, 3, 2)$

D. $(2, 3, 9)$

E. $(1, 2, 3)$

13. The plane S passes through the points $(1,2,3)$, $(3,2,1)$ and $(-3,0,3)$. Which of the following is an equation for S?

A. $x + 2y + z = 0$

B. $x - 2y + z = 0$

C. $x - 2y + z = 5$

D. $x + 2y + z = 5$

E. $x - y + z = 5$

14. A particle starts at the origin with initial velocity $\vec{i} + \vec{j} - \vec{k}$. Its acceleration is $\vec{a}(t) = 6t\ \vec{i} + 2\ \vec{j} - 6t\ \vec{k}$. Find its position at $t = 1$.

A. $\frac{1}{6}\ \vec{i} + \frac{1}{2}\ \vec{j} + \frac{1}{3}\ \vec{k}$

B. $\frac{7}{6}\ \vec{i} + \frac{1}{2}\ \vec{j} - \frac{5}{6}\ \vec{k}$

C. $3\ \vec{i} + 3\ \vec{j} - 5\ \vec{k}$

D. $2\ \vec{i} + 2\ \vec{j} - 2\ \vec{k}$

E. $2\ \vec{i} + 2\ \vec{j}$

15. Find the arc length of the curve defined by $\vec{r}(t) = (\cos(t), \sin(t), 2t)$, $-\pi \le t \le \pi$.

A. π

B. 2π

C. $2\sqrt{3}\pi$

D. $2\sqrt{5}\pi$

E. $2\sqrt{7}\pi$

16. Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{j} - 3\mathbf{k}$. What is $\mathbf{Proj_v u} =$

A. $\mathbf{Proj_v u} = -\mathbf{j} + \mathbf{k}$

B. $\mathbf{Proj_v u} = -\frac{10}{13}\mathbf{j} + \frac{15}{13}\mathbf{k}$

C. $\mathbf{Proj_v u} = -10\mathbf{j} + 15\mathbf{k}$

D. $\mathbf{Proj_v u} = \frac{2}{\sqrt{13}}\mathbf{j} - \frac{3}{\sqrt{13}}\mathbf{k}$

E. $\mathbf{Proj_v u} = \frac{1}{11}\mathbf{i} + \frac{2}{11}\mathbf{j} - \frac{3}{11}\mathbf{k}$

17. Find the curvature of the curve defined by $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at $t = 2$. Recall: $\kappa = |\frac{dT}{ds}| = |\frac{dT}{dt}|/|\mathbf{v}|$

A. $\frac{3}{5}$

B. $\frac{3}{4}$

C. $\frac{3}{25}$

D. $\frac{9}{25}$

E. 9

18. **Find a vector $\mathbf{a} \neq \mathbf{0}$, and vectors $\mathbf{b}$ and $\mathbf{c}$ such that**

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \quad \textbf{but} \quad \mathbf{b} \neq \mathbf{c}.$$

(You need to specify a, b, c , and calculate $\mathbf{a} \cdot \mathbf{b}$)

Your Answer:
$\mathbf{a} =$ ____________________
$\mathbf{b} =$ ____________________
$\mathbf{c} =$ ____________________
$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ ____________________

19. **Find <u>all</u> the values of x, such that the following series <u>converges</u> (do not forget the end values)**

$$\sum_{n=1}^{\infty} \frac{x^n}{n}.$$

Your Answer: For x satisfying ____________________, the series converges.