MA 271 Vector Calculus
Fall 1999, Test Two

Instructor: Yip

- This test booklet has SIX QUESTIONS, totaling 60 points for the whole test. You have 50 minutes to do this test. **Plan your time well. Read the questions carefully.**
- This test is closed book and closed notes.
- (Any kind of) calculator is allowed. But you should **not** use it whenever it is possible (from the point of view of this class), i.e. your answers should be as **analytical** as possible.
- In order to get full credits, you need to give **correct** and **simplified** answers and explain in a **comprehensible way** how you arrive at them.
- You can use both sides of the papers to write your answers. But please indicate so if you do.

Name:____

Question	Score
1.(10 pts)	
2.(10 pts)	
3.(10 pts)	
4.(10 pts)	
5.(10 pts)	
6.(10 pts)	
Total (60 pts)	

1. Write down a series (consisting of a finite number of terms) that will give the value of the following integral up to 10^{-16} accuracy:

$$\int_0^1 e^{x^2}\, dx$$

2. What are the following limits:

$$\lim_{t \to 0} \frac{1 - \cos(t^2) - \frac{t^4}{2}}{t^6} \tag{1}$$

$$\lim_{x \to \infty} x^2 \left(e^{-\frac{1}{x^2}} - 1 \right) \tag{2}$$

3. Compute the arc length of the following curve which is given by $r = 1 + \cos\theta$ in terms of the polar coordinates:

4. An object is given in terms of the spherical coordinates by

$$\rho = \cos\phi$$

(a) Express the above object in terms of the cartesian coordinates (x, y, z);

(b) *Identity* and *plot out* the above object in $x - y - z$ space. *Label important* points.

5. Plot out the 0 and -1 *level surfaces* of the following function:

$$w = f(x, y, z) = z + x^2 + y^2$$

6. You are given the following function:

$$z = f(x, y) = \frac{1}{1 + x^2 + y^2}$$

(a) Plot out the graph in the $x - y - z$ space;

(b) Find out the equation of the tangent plane to f at $(x = 1, y = 2)$.

Taylor Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \cdots \frac{f^{(n)}(x-a)^n}{n!} + \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$$

(where c is in between x and a)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \qquad |x| < 1$$

Trigonemetric Formulas

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha;$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta;$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha;$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha;$$

Polar Coordinates **Cylindrical Coordinate** **Spherical Coordinates**

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$