Math 385 | Midterm Exam | March 10, 2004

NAME SOLUTIONS

(10 pts) **1.** Translate each of the following sentences into the language of the Sentential Calculus using the indicated letters as abbreviations .

(a) Oscar will pass the logic course only if he studies. (P, S)

$$P \to S$$

(b) Oscar will not pass the logic course if he neither does his homework nor studies. (P, H, S).

$$(\sim H \wedge \sim S) \to \sim P$$

or $$P \to (H \vee S)$$

or $$\sim P \vee S \vee H$$

(c) It is not the case that Oscar will pass the logic course provided that he studies and does his homework. (P, H, S)

$$\sim((S \wedge H) \to P)$$

or $$S \wedge H \wedge \sim P$$

(d) If I miss my train I will arrive 10 minutes late, assuming the next train is on time. (M, L, N)

$$(N \wedge M) \to L$$

(e) If two lines lie in a plane then a necessary and sufficient condition for them to be parallel is that they neither intersect nor coincide. (L, P, I, C).

$$L \to (P \leftrightarrow (\sim I \wedge \sim C))$$

(10 pts) 2. (a) Using a truth table, or otherwise, find a sentence in disjunctive normal form which is logically equivalent to the sentence

$$((P \vee \neg Q) \to R) \leftrightarrow (Q \wedge \neg R).$$

P	Q	R	$(P \vee \neg Q)$	$\to R$	$Q \wedge \neg R$	
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	T	T	F	F
T	F	F	T	F	F	T
F	T	T	F	T	F	F
F	T	F	F	T	T	T
F	F	T	T	T	F	F
F	F	F	T	F	F	T

$P\bar{Q}\bar{R} \vee \bar{P}Q\bar{R} \vee \bar{P}\bar{Q}\bar{R}$

[Replace $\bar{P}\bar{Q}\bar{R}$ by $\bar{P}\bar{Q}\bar{R} \vee \bar{P}\bar{Q}\bar{R}$ to get $\bar{P}\bar{R} \vee \bar{Q}\bar{R}$ or $\overline{PQ}\,\bar{R}$)]

(b) Find a sentence in conjunctive normal form which is logically equivalent to the above sentence.

$(\bar{P} \vee \bar{Q}) \wedge \bar{R}$ (see above)

(10 pts) **3.** Construct a formal proof, as in Fitch, for the following argument. Number the lines in the proof, and wherever justification by Rules is usually called for, just use the line numbers to indicate which lines and subproofs you would quote (without explicitly naming the appropriate Rule).

Premises: $A \vee (B \to C)$; $C \to D$; $(B \to D) \to E$.

Conclusion: $\neg A \to E$.

Note. If you get hung up on this, you can still get partial credit for getting part of the proof right. And do leave yourself enough time for the next page!

1	$A \vee (B \to C)$	
2	$C \to D$	
3	$(B \to D) \to E$	
4	$\neg A$	
5	B	
6	A	
7	$\perp$	(4,6)
8	D	(7)
9	$B \to C$	
10	C	(5,9)
11	D	(2,10)
12	D	(1, 6-8, 9-11)
13	$B \to D$	(5-12)
14	E	(3, 13)
15	$\neg A \to E$	(4-14)

(10 pts) **4.** At a murder trial, four witnesses testify as follows:

1. Either Arthur is guilty or if Betty is innocent then Charles is innocent.
2. If Charles is innocent then Dave is innocent.
3. If either Betty is guilty or Dave is innocent then Edward is innocent.
4. Arthur is innocent and Edward is guilty.

Are these statements consistent with each other? In other words, is it possible that all four witnesses are telling the truth?

Justify your answer.

<u>Hint</u>. You may assume, if you wish, that the conclusion stated on page 3 is a consequence of the premises on that page, even if you haven't proved it.

Write A for "Arthur guilty"
B for "Betty innocent"
C for "Charles innocent"
D for "Dave innocent"
E for "Edward innocent"

Then 1, 2, 3 above are the same as 1, 2, 3 on previous page (the premises), and 4 above is the <u>negation</u> of the conclusion from these premises: $\sim(\sim A \to E) \Leftrightarrow \sim(A \vee E) \Leftrightarrow (\sim A \wedge \sim E)$.

So 1, 2, 3, 4 above are, taken together, inconsistent (again, because 1, 2, and 3 imply the negation of 4).