

A VANISHING THEOREM FOR FINITELY SUPPORTED IDEALS IN REGULAR LOCAL RINGS: RETRACTION

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I am grateful to Shihoko Ishii for pointing out an overreach in the central result, Theorem 2.1, of [L08].¹ Most of the proofs in [L08] use that theorem, so are hereby *recanted*.

In [I26], Ishii proves some new cases of the following scaled-down version (already known when $\dim R = 2$ or when $R/\mathfrak{m}$ has characteristic 0):

Let R be a regular local ring of dimension $d \geq 2$, with maximal ideal $\mathfrak{m}$, and f a composite scheme-map

$$X = X_r \xrightarrow{f_r} X_{r-1} \xrightarrow{f_{r-1}} \cdots \xrightarrow{f_1} X_0 = \operatorname{Spec}(R) \quad (r > 0)$$

where for $1 \leq i \leq r$, f_i is the blowup of a closed point in X_{i-1} . Let $E = f^{-1}\mathfrak{m}$, let $\omega_f \cong H^0 f^! \mathcal{O}_{X_0}$, and let I be an R -ideal such that $I\mathcal{O}_X$ is an invertible $\mathcal{O}_X$ -module. Then for all $i \neq 0$, one has $H^i(X, I\omega_f) = 0$.

Remarks. 1. The relative dualizing $\mathcal{O}_X$ -module ω_f can be taken to be J_f^{-1} , where $J_f = J_{f_r} J_{f_{r-1}} \cdots J_{f_1}$ is the Jacobian ideal of f . (Cf. [LS81, (1.1) and (2.3)], whose proofs, though local, are easily globalized.) The calculation at the top of p. 202 in [LS81] implies that $J_{f_1} = \mathfrak{m}^{d-1} \mathcal{O}_{X_1}$ —whence, by induction on r , J_f is invertible.

2. The R -ideal I can be replaced by $a^{-1}I$ with a a g.c.d. of the members of I , so I can be assumed to contain a power of $\mathfrak{m}$.

REFERENCES

- [I26] S. Ishii, Kawamata-Viehweg and Lipman’s vanishings for a sequence of blow-ups by regular closed points, arxiv:2610.26518.
- [L08] ———, A vanishing theorem for finitely supported ideals in regular local rings, *Michigan Math. J.* **57** (2008), 573–585.
- [LS81] ———, A. Sathaye, Jacobian ideals and a theorem of Briançon-Skoda, *Michigan Math. J.* **28** (1981), 199–222.

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¹Her observations: With notation as in *loc. cit.*, $d = 2$, $r = 2$, f_2 the blowup of $p \in E_0 := f_1^{-1}\mathfrak{m}$, $E_1 \subset X_2$ the proper transform of E_0 , $E_2 := f_2^{-1}p$ —so that $K_f = 2E_2 + E_1$ —and $D := K_f + 2E_2$, [L08, Corollary 2.4(i)] asserts that $H^1(X, \mathcal{O}_X(-2E_2)) = 0$. But with $\mathcal{M}_p$ the $\mathcal{O}_{X_1}$ -ideal such that the stalk of $\mathcal{O}_{X_1}/\mathcal{M}_p$ vanishes everywhere except at p , where it is the residue field, one has

$$H^1(X, \mathcal{O}_X(-2E_2)) = H^0(X_0, Rf_{1*} Rf_{2*} \mathcal{O}_X(-2E_2)) = H^0(X_0, Rf_{1*} \mathcal{M}_p^2) \neq 0.$$

This discrepancy is due to the *erroneous claim* in the proof of [L08, Lemma 2.2], that the last square in its diagram always commutes, a mistake that vitiates Corollary 2.4.