Corrections to

Local Homology and Cohomology on Schemes

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1. In the proof of Lemma (5.3.3)(b), the "way-out" reduction requires a boundedness restriction on the functor $\mathbf{R}\mathcal{H}om^{\bullet}_{\widehat{X}}(\kappa^*\mathcal{E}, \kappa^*\mathcal{F})$. That this restriction holds can be shown as in [DFS, p. 23].

The argument associated with (2) on p. 8 suffers from a similar—though more easily remedied—deficiency.

2. The paragraph preceding (1.2) says more than it has a right to, and so for non-affine X the left-derived functor $\mathbf{L}\Lambda_Z$ is not always defined on *all* of $\mathbf{D}_{\mathrm{qc}}(X)$, but rather on an equivalent subcategory. This problem is dealt with in [GM, p. 95] (see second paragraph there).

3. The most serious error, kindly pointed out by Peter Schenzel (August, 2000), occurs in the proof of Lemma (3.1.1).

What that overly-concise proof actually shows is that condition (1) (proregularity of $\mathbf{t}$) is equivalent to all the other conditions holding not only for the sequence $\mathbf{t}$ but also *for every initial segment* $(t_1, \dots, t_\nu)$ $(\nu < \mu)$.

(For example, in $\prod_{n>0} \mathbb{Z}/2^n\mathbb{Z}$, if $t := (2, 2, 2, \dots)$ then the sequence $(1, t)$ is proregular but $(t, 1)$ is not. Thus the implication $(2) \Rightarrow (1)$ can fail.)

Consequently the notion of "proregular" must be redefined in order for subsequent results to be valid. One way to repair the damage is as follows.

Call $\mathbf{t}$ *weakly proregular* if one of the conditions (2), (2)$'$, (3) or (3)$'$ of Lemma (3.1.1) (all of which are shown by the proof to be equivalent) hold. Substitute "weakly proregular" for "proregular" everywhere in Section 3, including the definition of proregularly embedded subspace, but excluding Definition (3.0.1)—which might be recharacterized as "total proregularity." (As mentioned above, it is equivalent to weak proregularity of all of the initial segments of $\mathbf{t}$.) Then all the results are valid, with the following minor modifications.

• (A propos of Examples (a) and (b) in §(3.0)—which still apply to total proregularity.) Lemma(3.1.1)(2) shows that weak proregularity of a sequence is a *local* condition: it holds on X iff it holds over each member of an open cover of X. In fact Lemma(3.1.1)(3)$'$ shows that weak proregularity of a sequence is preserved under all *flat* ringed-space maps. In particular, if it holds for $\mathbf{t}$ on X then it holds for $\mathbf{t}$ in $\mathcal{O}_x$ for all $x \in X$; and the converse obtains if stalks of injective sheaves are injective, for example if $\mathcal{O}$ is coherent and $\mathcal{O}_x$ is noetherian for all x [Gr, p. 187, Prop. 4.1.2].

• In Lemma (3.1.1), condition (3)$''$ will no longer be equivalent to the others.

• In Corollary (3.1.3) one must *assume further* that $\mathbf{t}$ is weakly proregular.

Now with notation as in Definition (3.0.1), for the pivotal results in §4—hence for §5—to remain valid, it suffices to define $\mathbf{t}$ *proregular* to mean that $\mathbf{t}$ is weakly proregular and that in addition there is an integer N such that, in $\mathcal{O}$, we have $(0)\colon t_i^N = (0)\colon t_i^{N+1}$ for all $i = 1, 2, \dots, \mu$. (In other words each one-element subsequence (t_i) is weakly proregular.)

References

[DFS] L. Alonso Tarrío, A. Jeremías López and J. Lipman, *Duality and flat base change on formal schemes*, Contemp. Math. **244** (1999), 3-90.

[GM] ———, *Greenlees-May Duality on formal schemes*, Contemp. Math. **244** (1999), 93–112.

[Gr] A. Grothendieck, *Quelques points d'algèbre homologique*, Tôhoku Math. J **9** (1957), 119–221.