PRACTICE EXAM 6

1. A survey of the public determines the following about the "Lord of the Rings" trilogy (3 movies).

Have Seen #1	Have Seen #2	Have Seen #3	Percentage of Public
No	No	No	50%
Yes	?	?	35%
?	Yes	?	33%
?	?	Yes	31%
Yes	No	No	8%
Yes	Yes	No	4%
Yes	Yes	Yes	20%

Based on this information, determine the percentage of the public that has seen exactly one of the three "Lord of the Rings" movies.

A) 15 B) 17 C) 19 D) 21 E) 23

2. Suppose that events A and B are independent and suppose that $A \subseteq B$. Which of the following pairs of values is impossible?

A) $P(A) = \frac{1}{3}$ and $P(B) = 1$ B) $P(A) = \frac{1}{2}$ and $P(B) = 1$

C) $P(A) = 0$ and $P(B) = \frac{1}{2}$ D) $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$

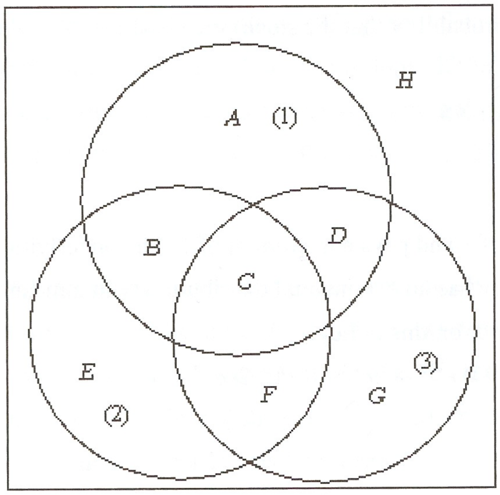
E) $P(A) = 1$ and $P(B) = 1$

3. For a particular disease, it is found that 1% of the population will develop the disease and 2% of the population has a family history of having the disease. A genetic test is devised to predict whether or not the individual will develop the disease. For those with a family history of the disease, 20% of the time the genetic test predicts that the individual will develop the disease and for those with no family history of the disease, 1% of the time the genetic test predicts that the individual will develop the disease. The genetic test is not perfect, and individuals are followed to determine whether or not they actually develop the disease. It is found that for those who have a family history of the disease and for whom the genetic test predicts the disease will develop, 80% actually develop the disease. It is also found that for those who have a family history of the disease and for whom the genetic test does not predict the disease will develop, 10% actually develop the disease. Find the probability that someone with a family history of the disease will develop the disease.

A) .20 B) .22 C) .24 D) .26 E) .28

PRACTICE EXAM 6 - SOLUTIONS

1. We can represent the events in the following diagram:



The top circle, $A \cup B \cup C \cup D$ represents the event of having seen #1 of the movie series, the lower left circle, $E \cup B \cup C \cup F$ represents the event of having seen #2 of the movie series, the lower right circle, $G \cup F \cup C \cup D$ represents the event of having seen #3 of the movie series, and H represents the event of having seen none of the three movies.

From the given information, we know that the percentage for event H is $h = 50$.
The second line of the information table indicates that 35% of the public has seen movie #1 but we don't know about movies #2 and #3 for this group. This is interpreted as the percentage for $A \cup B \cup C \cup D$ is $a + b + c + d = 35$.

Similarly, the percentage for $E \cup B \cup C \cup F$ is $e + b + c + f = 33$, and the percentage for $G \cup F \cup C \cup D$ is $g + f + c + d = 31$.
The 5th line of the table indicates that 8% have seen movie #1 and not movies #2 or #3. Therefore, the percentage for event A is $a = 8$.
Event B is the event of having seen both #1 and #2 but not #3 and this has percentage $b = 4$, and event C is the event of have seen all three, and this has percentage $c = 20$.

1. continued

The event of having seen exactly one of the three movies is the combination $A \cup E \cup G$.

This will be $a+e+g$.

We know that $a+b+c+d+e+f+g+h=100$ percent, since everyone either sees a movie or doesn't. This leads to the following 8 equations:

$h=50\,(1)$, $a+b+c+d=35\,(2)$, $e+b+c+f=33\,(3)$, $g+f+c+d=31\,(4)$, $a=8\,(5)$, $b=4\,(6)$, $c=20\,(7)$, $a+b+c+d+e+f+g+h=1\,(8)$.

From equations (3), (6) and (7) we get $e+f=9\,(9)$.

From equations (1), (2) and (8) we get $e+f+g=15\,(10)$.

From equations (9) and (10) we get $g=6\,(11)$.

From equations (2), (5) and (6) we get $c+d=23\,(12)$.

From equations (11), (12) and (4) we get $f=2\,(13)$.

From equations (9) and (13) we get $e=7$.

Then $a+e+g=8+7+6=21$ is the percentage that has seen exactly one of the three movies.

Once we have determined the individual values of a,b,c,d,e,f,g,h , we can find the percentage for any combination. For instance, the percentage of people who have seen #1 and #3 but not #2 is $d=3$.   Answer: D

2. Since A and B are independent, we must have $P(A \cap B) = P(A) \cdot P(B)$.

Since $A \subseteq B$, it follows that $A \cap B = A$.

Therefore, we must have $P(A) = P(A) \cdot P(B)$.

The only way this can be true is if $P(A)$ is 0 or 1 , or if $P(B)$ is 1.

Only D does not satisfy one of these conditions.   Answer: D