HW 9 #15 $\quad g(x) = \underbrace{7x^3}_{f(x)} \underbrace{\cot x}_{h(x)} \qquad g'(\pi/2)$

$$g'(x) = f'(x)\,h(x) + f(x)\,h'(x) \qquad \text{(product rule)}$$

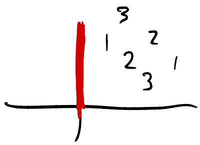
$f(x) = 7x^3$

$f'(x) = 7 \cdot 3x^2$

$= 21x^2$

$h(x) = \cot x$

$h'(x) = -\csc^2 x$



$$g'(x) = (21x^2)\cot x + (7x^3)(-\csc^2 x)$$

$$= 21x^2 \cot x - 7x^3 \csc^2 x$$

$$g'\left(\tfrac{\pi}{2}\right) = 21\left(\tfrac{\pi}{2}\right)^2 \cot\left(\tfrac{\pi}{2}\right) - 7\left(\tfrac{\pi}{2}\right)^3 \csc^2\left(\tfrac{\pi}{2}\right)$$

$$= 21 \cdot \frac{\pi^2}{4} \cdot 0 - 7\frac{\pi^3}{8}(1)^2$$

$$= 0 - 7\pi^3/8$$

$\tan\left(\frac{\pi}{2}\right) \overset{?}{=} \infty$

$\cot\left(\frac{\pi}{2}\right) = 0$

$\csc x = \dfrac{1}{\sin x}$

$\csc\frac{\pi}{2} = \dfrac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1$

Lesson 10: Chain rule

Suppose $y = g(h(x))$ what is dy/dx?

g is the outside function

h is the inside function

Chain rule

$$\frac{dy}{dx} = \underline{\frac{dy}{dh}} \cdot \underline{\frac{dh}{dx}}, \qquad y' = \underline{g'(h(x))} \cdot \underline{h'(x)}$$

derivative of outside fun. w.r.t. to inside fun.

times derivative of inside fun w.r.t. x

Examples

① $f(x) = 6(x^2+1)^{2022}$

$g(x) = 6x^{2022}$ $\qquad f(x) = g(h(x))$

$h(x) = x^2+1$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$g'(x) = 6 \cdot 2022\, x^{2021}$

$h'(x) = 2x$

$$f'(x) = 6 \cdot 2022 (x^2+1)^{2021} \cdot 2x$$

$$= 24264\, x (x^2+1)^{2021}$$

② $f(x) = \sqrt[5]{x^3+3x-1}$

$g(x) = \sqrt[5]{x} = x^{\frac{1}{5}}$ $\qquad h(x) = x^3+3x-1$

$g'(x) = \frac{1}{5} x^{-4/5}$ $\qquad h'(x) = 3x^2+3$

$$g(h(x)) = \sqrt[5]{h(x)} = \sqrt[5]{x^3+3x-1}$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$= \frac{1}{5}(x^3+3x-1)^{-4/5} \cdot (3x^2+3)$$

(3) $y = \left(\dfrac{8x^2}{x+1} \right)^3$

$g(x) = x^3$ $\quad$ $h(x) = \dfrac{8x^2}{x+1}$

$g'(x) = 3x^2$ $\quad$ $h'(x) = \dfrac{16x(x+1) - 8x^2(1)}{(x+1)^2}$

$$y' = g'(h(x)) \cdot h'(x)$$

$$= 3\left(\frac{8x^2}{x+1}\right)^2 \cdot \frac{16x(x+1) - 8x^2}{(x+1)^2} \quad \checkmark$$

$$= \frac{3 \cdot (8x^2)^2 (16x(x+1) - 8x^2)}{(x+1)^4}$$

(4) $f(x) = (e^x + \sin x)^{2/3}$

$g(x) = x^{2/3}$ $\quad$ $h(x) = e^x + \sin x$

$g'(x) = \frac{2}{3} x^{-1/3}$ $\quad$ $h'(x) = e^x + \cos x$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$= \frac{2}{3}(e^x + \sin x)^{-1/3} \cdot (e^x + \cos x)$$

⑤ $g(x) = -(x^3+1-e^x)^7$

outside: $f(x) = -x^7$ inside: $h(x) = x^3+1-e^x$

$f'(x) = -7x^6$ $h'(x) = 3x^2 - e^x$

$$g'(x) = f'(h(x)) \cdot h'(x)$$

$$= -7(x^3+1-e^x)^6 \cdot (3x^2 - e^x) \quad \checkmark$$

HW 9 # 4

$$g(x) = \frac{6\sin x - 6\cos x}{\sin x + \cos x} \begin{matrix} = f(x) \\ = h(x) \end{matrix}, \quad g'(x)$$

$$g'(x) = \frac{f'(x)h(x) - f(x)h'(x)}{(h(x))^2}$$

$f(x) = 6\sin x - 6\cos x$

$f'(x) = 6\cos x - 6(-\sin x)$

$= 6\cos x + 6\sin x$

$h(x) = \sin x + \cos x$

$h'(x) = \cos x - \sin x$

$$g'(x) = \frac{(6\cos x + 6\sin x)(\sin x + \cos x) - (6\sin x - 6\cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{6\cos x\sin x + 6\cos^2 x + 6\sin^2 x + 6\sin x\cos x - (6\sin x\cos x - 6\sin^2 x - 6\cos^2 x + 6\cos x\sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\cancel{6\sin x\cos x} + 6\cos^2 x + 6\sin^2 x + \cancel{6\sin x\cos x} - \cancel{6\sin x\cos x} + 6\sin^2 x + 6\cos^2 x - \cancel{6\sin x\cos x}}{(\sin x + \cos x)^2}$$

$$= \frac{12(\overbrace{\sin^2 x + \cos^2 x}^{=1})}{(\sin x + \cos x)^2} = \frac{12}{(\sin x + \cos x)^2}$$

Lesson 10: Chain rule

Suppose $y = g(h(x))$. What is dy/dx?

We say g is the outside function

h is the inside function

Chain rule

$$\frac{dy}{dx} = \frac{dy}{dh} \cdot \frac{dh}{dx} \qquad y' = g'(h(x)) \cdot h'(x)$$

Examples

① $f(x) = 6(x^2+1)^{2022}$

$g(x) = 6x^{2022}$ $\quad h(x) = x^2+1$

$g'(x) = 6 \cdot 2022\, x^{2021}$ $\quad h'(x) = 2x$

$$g(h(x)) = 6(h(x))^{2022}$$

$$= 6(x^2+1)^{2022} = y \checkmark$$

$$y' = g'(h(x)) \cdot h'(x)$$

$$= 6 \cdot 2022 (h(x))^{2021} \cdot h'(x)$$

$$= 6 \cdot 2022 (x^2+1)^{2021} \cdot 2x$$

$$= 2 \cdot 6 \cdot 2022\, x (x^2+1)^{2021}$$

(2) $f(x) = \sqrt[5]{x^3 + 3x - 1}$

$g(x) = \sqrt[5]{x} = x^{1/5}$ $\qquad h(x) = x^3 + 3x - 1$

$g'(x) = \frac{1}{5} x^{-4/5}$ $\qquad h'(x) = 3x^2 + 3$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$= \frac{1}{5}(x^3 + 3x - 1)^{-4/5} \cdot (3x^2+3)$$

$$= \frac{3x^2+3}{5\sqrt[5]{(x^3+3x-1)^4}}$$

(3) $y = \left(\frac{8x^2}{x+1}\right)^3$

$g(x) = x^3$ $\qquad h(x) = \frac{8x^2}{x+1}$

$g'(x) = 3x^2$ $\qquad h'(x) = \frac{16x(x+1) - 8x^2(1)}{(x+1)^2}$

$$= \frac{16x^2 + 16x - 8x^2}{(x+1)^2}$$

$$= \frac{8x^2 + 16x}{(x+1)^2}$$

$$y' = g'(h(x)) \cdot h'(x)$$

$$= 3\left(\frac{8x^2}{x+1}\right)^2 \cdot \frac{8x^2+16x}{(x+1)^2}$$

④ $f(x) = \left(e^x + \sin x\right)^{2/3}$

$g(x) = x^{2/3}$ $\qquad h(x) = e^x + \sin x$

$g'(x) = \frac{2}{3}x^{-1/3}$ $\qquad h'(x) = e^x + \cos x$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$= \frac{2}{3}\left(e^x + \sin x\right)^{-1/3} \cdot \left(e^x + \cos x\right)$$

⑤ $y = -\left(x^3 + 1 - e^x\right)^7$

$g(x) = -x^7$ $\qquad h(x) = x^3 + 1 - e^x$

$g'(x) = -7x^6$ $\qquad h'(x) = 3x^2 - e^x$

$$y' = g'(h(x)) \cdot h'(x)$$

$$= -7\left(x^3 + 1 - e^x\right)^6 \left(3x^2 - e^x\right)$$