Lesson 12: Higher order derivatives

The second derivative

$$\frac{d^2}{dx^2}\left[f(x)\right] = \frac{d}{dx}\left[\frac{d}{dx}\left(f(x)\right)\right]$$

The nth derivative

$$\frac{d^n}{dx^n}\left(f(x)\right) = \underbrace{\frac{d}{dx}\left(\cdots \frac{d}{dx}\right.}_{n \text{ derivatives}}\left.\left(f(x)\right)\cdots\right)$$

$f^{(n)}(x)$

$f'''(x)$: the third derivative

$f^{(5)}(x)$: the 5th derivative

Examples

(1) $y = x^3 + 2x$ find y''.

$y' = 3x^2 + 2$ first derivative

$y'' = 6x + 0$ second derivative

(2) $g(x) = \underbrace{6e^{5x}}\,\underbrace{\cos(2x)}$ find d^2g/dx^2

$$\frac{dg}{dx} = \frac{d}{dx}\left[6e^{5x}\right]\cos(2x) + 6e^{5x}\frac{d}{dx}\left[\cos(2x)\right]$$

$$\frac{d}{dx}\left[6e^{5x}\right] =$$

$$\parallel$$

out : $g(x) = 6e^{x}$ $\quad g'(x) = 6e^{x}$

in : $h(x) = 5x$ $\quad h'(x) = 5$

$$6e^{(5x)}\cdot 5 = 30e^{5x}$$

$$\frac{d}{dx}\left[\cos(2x)\right] \qquad \text{out: } g(x) = \cos x \quad g'(x) = -\sin x$$
$$\text{in: } h(x) = 2x \quad h'(x) = 2$$

$$= -\sin(2x)\cdot 2 = -2\sin(2x)$$

$$\frac{dy}{dx} = \left(30e^{5x}\right)\left(\cos(2x)\right) + \left(6e^{5x}\right)\left(-2\sin(2x)\right)$$

$$= 30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[30e^{5x}\right]\cos(2x) + 30e^{5x}\frac{d}{dx}\left(\cos(2x)\right)$$
$$+ \frac{d}{dx}\left[-12e^{5x}\right]\sin(2x) - 12e^{5x}\frac{d}{dx}\left(\sin(2x)\right)$$

$$= 30\cdot 5e^{5x}\cos(2x) + 30e^{5x}\left(-2\sin(2x)\right)$$
$$- 12\cdot 5e^{5x}\sin(2x) - 12e^{5x}\cos(2x)\cdot 2$$

$$= 150e^{5x}\cos(2x) - 60e^{5x}\sin(2x) - 60e^{5x}\sin(2x)$$
$$- 24e^{5x}\cos(2x)$$

$$= 126e^{5x}\cos(2x) - 120e^{5x}\sin(2x)$$

(3) Let $s(t) = t^3 + 10t^2 - t + 1$ be the position of an obj w.r.t. to time t.
(meters) (seconds)

Find the acceleration.

$$a(t) := \text{R.o.C of velocity}$$
$$:= \text{R.o.C. of R.o.C of postion}$$
$$:= s''(t)$$

$$v(t) = s'(t) = 3t^2 + 20t - 1 \quad \text{Velocity}$$
$$a(t) = s''(t) = v'(t) = 6t + 20 \quad \text{acceleration}$$

(4) Suppose that $f^{(4)}(x) = 10\sec(2x)$ find $f^{(5)}(x)$.

$$f^{(5)}(x) = \frac{d}{dx}\left(f^{(4)}(x)\right) = \frac{d}{dx}\left(10\sec(2x)\right)$$
$$= \overbrace{10\sec(2x)\tan(2x)} \cdot 2$$
$$= 20\sec 2x \tan 2x$$

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The second derivative

$$\frac{d^2}{dx^2}\left(f(x)\right) := \frac{d}{dx}\left(\frac{d}{dx}\left(f(x)\right)\right)$$

$$f''(x) \quad , \quad f^{(2)}(x)$$

$$f'''(x) \quad , \quad f^{(3)} \quad , \quad \frac{d^3}{dx^3} \qquad \text{third derivative}$$

$$\frac{d}{dx}\left[\frac{d}{dx}\left[\frac{d}{dx}\left[f(x)\right]\right]\right]$$

The nth derivative

$$\frac{d^n}{dx^n}\left(f(x)\right) := \underbrace{\frac{d}{dx}\left(\cdots \frac{d}{dx}\right.}_{n \text{ derivatives}}\left.\left(f(x)\right)\cdots\right)$$

Example

① $y = x^3 + 2x$ find y''

$$y' = 3x^2 + 2$$

$$y'' = 6x + 0 = 6x$$

(2) $g(x) = 6e^{5x}\cos(2x)$ find d^2g/dx^2.

$$\frac{dg}{dx} = \frac{d}{dx}\left(6e^{5x}\right)\cos(2x) + 6e^{5x}\frac{d}{dx}\left(\cos(2x)\right)$$

$$\frac{d}{dx}\left(6e^{5x}\right)$$

out: $g(x) = 6e^{x}$ $\quad g'(x) = 6e^{x}$

in: $h(x) = 5x$ $\quad h'(x) = 5$

$$= 6e^{(5x)} \cdot 5 = 30e^{5x}$$

$$\frac{d}{dx}\left(\cos(2x)\right)$$

out: $g(x) = \cos x$ $\quad g'(x) = -\sin x$

in: $h(x) = 2x$ $\quad h'(x) = 2$

$$= -\sin(2x)\cdot 2 = -2\sin(2x)$$

$$\frac{dg}{dx} = 30e^{5x}\cdot\cos(2x) + 6e^{5x}\left(-2\sin(2x)\right)$$

$$= 30e^{5x}\cos(2x) - 12e^{5x}\sin(2x)$$

$$\frac{d^2g}{dx^2} = \frac{d}{dx}\left(30e^{5x}\right)\cos(2x) + 30e^{5x}\frac{d}{dx}\left(\cos(2x)\right)$$

$$+ \frac{d}{dx}\left(-12e^{5x}\right)\sin(2x) - 12e^{5x}\frac{d}{dx}\left(\sin 2x\right)$$

$$= 30\cdot 5e^{5x}\cos 2x + 30e^{5x}\left(-2\sin 2x\right)$$

$$-12\cdot 5e^{5x}\sin 2x - 12e^{5x}\,2\cdot\cos 2x$$

$$= 126e^{5x}\cos(2x) - 120e^{5x}\sin(2x) \quad \checkmark$$

(3) Let $s(t) = t^3 + 10t^2 - t + 1$ be the position of an obj (meters) w.r.t to time t. Find acceleration of the obj.

$a(t) =$ R.o.C of velocity.

$=$ R.o.C. of R.o.C of position

$= s''(t)$

$a(t) = v'(t) = s''(t)$

$v(t) = s'(t) = 3t^2 + 20t - 1$

$a(t) = s''(t) = 6t + 20$ ✓

(4) Suppose $f^{(4)}(x) = 10\sec(2x)$. find $f^{(5)}(x)$.

$$f^{(5)}(x) = \frac{d}{dx}\left(f^{(4)}(x)\right) = \frac{d}{dx}\left(10\sec(2x)\right)$$

$$= 10\sec(2x)\tan(2x) \cdot 2$$

$$= 20\sec(2x)\tan(2x).$$