Lesson 13: Implicit differentiation

Explicit functions : $y = 3x$ $\quad f(x) = 6\cos(2x) + 1$ ✓

Implicit function : $f(x,y) = 0$.

Examples

① $y - 3x = 0$

② $3y^2 = 1 + 5x^2$

③ $\sin(x + 3y) = 2xy$

④ $e^{xy} = 2x$

① $y - 3x = 0$. $\quad dy/dx = ?$

$$\frac{d}{dx}[y - 3x] = \frac{d}{dx}[0] \qquad \frac{dy}{dx} = 3$$

$$\frac{dy}{dx} - 3\frac{d}{dx}[x] = 0$$

$$\frac{dy}{dx} - 3 = 0$$

$$y - 3x = 0$$
$$y = 3x$$
$$\frac{dy}{dx} = 3$$ ✓

② find equation of tan. line to $2x^4 = 4y^2 + 6x^2$ at $(2, \sqrt{2})$.

$$y - y_1 = m(x - x_1)$$

m : slope = derivative at $(2, \sqrt{2})$ $\quad \boxed{m = \left.\frac{dy}{dx}\right|_{(2,\sqrt{2})}}$

(x_1, y_1) : point on the line $(2, \sqrt{2})$ ✓

$$\frac{d}{dx}[2x^4] = \frac{d}{dx}[4y^2 + 6x^2]$$

$$8x^3 = 4\frac{d}{dx}(y^2) + 12x$$

$$8x^3 = 4 \cdot 2(y)^{2-1} \cdot \frac{dy}{dx} + 12x$$

out : $g(u) = u^2$ $\quad 2(y)^1 \cdot \frac{dy}{dx}$

in : y

$$8x^3 - 12x = 8y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{8x^3 - 12x}{8y}$$

$$\left.\frac{dy}{dx}\right|_{(\underset{x}{2},\, \underset{y}{\sqrt{2}})} = \frac{8 \cdot 2^3 - 12 \cdot 2}{8\sqrt{2}} = \frac{64 - 24}{8\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$y - \sqrt{2} = \frac{5}{\sqrt{2}}(x - 2) \checkmark$$

(3) $\sin(x + 3y) = 2xy$ find dy/dx

$$\frac{d}{dx}\left[\sin(x + 3y)\right] = \frac{d}{dx}\left[2xy\right]$$

$$\cos(x + 3y)\left(1 + 3\frac{dy}{dx}\right) = 2y + 2x\frac{dy}{dx}$$

$$\cos(x + 3y) + 3\cos(x + 3y)\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$\cos(x + 3y) - 2y = -3\cos(x + 3y)\frac{dy}{dx} + 2x\frac{dy}{dx}$$

$$\cos(x+3y) - 2y = \left(-3\cos(x+3y) + 2x\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos(x+3y) - 2y}{-3\cos(x+3y) + 2x}$$ ✓

④ $e^{xy} = 2x$ find dy/dx

$$\frac{d}{dx}\left[e^{xy}\right] = \frac{d}{dx}\left[2x\right]$$

out: $g(u) = e^u$

in: xy

$$\frac{d}{dx}(xy) = 1\cdot y + x\frac{dy}{dx}$$

$$e^{xy}\left(y + x\frac{dy}{dx}\right) = 2$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = 2$$

$$xe^{xy}\frac{dy}{dx} = 2 - ye^{xy}$$

$$\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy}}$$ ✓

⑤ $\tan(x/y) = 10x$, find dy/dx

$$\frac{d}{dx}\left[\tan(x/y)\right] = \frac{d}{dx}\left[10x\right]$$

$$\sec^2\left(\frac{x}{y}\right)\cdot\frac{d}{dx}\left[\frac{x}{y}\right] = 10$$

$$\frac{d}{dx}\left[\frac{x}{y}\right] = \frac{1\cdot y - x\cdot dy/dx}{y^2}$$

$$\sec^2\left(\frac{x}{y}\right)\cdot\frac{y - x\ dy/dx}{y^2} = 10$$

$$\frac{y - x\ dy/dx}{y^2} = \frac{10}{\sec^2(x/y)}$$

$$\frac{y - x\ dy/dx}{y^2} = 10\cos^2(x/y)$$

$$y - x\frac{dy}{dx} = 10y^2\cos^2(x/y)$$

$$-x\frac{dy}{dx} = 10y^2\cos^2(x/y) - y$$

$$\frac{dy}{dx} = \frac{y - 10y^2\cos^2(x/y)}{x} \quad \checkmark$$

(6) $5\cos x \sin y = 1$ find dy/dx

$$\frac{d}{dx}\left[\underbrace{5\cos x}\ \underbrace{\sin y}\right] = \frac{d}{dx}[1] = 0$$

$$-5\sin x\sin y + 5\cos x\cdot\frac{d}{dx}[\sin y] = 0$$

$$-5\sin x\sin y + 5\cos x\cdot\cos(y)\cdot\frac{dy}{dx} = 0$$

HW 12 #6 particle traveling on a line

position : $s(t) = \frac{11}{3}t^3 + 187t^2$

a) What is acc.?

R.o.C = derivative

acc. $a(t)$ = R.o.C. of velocity

vel. $v(t)$ = R.o.C. of position.

$$v(t) = s'(t) = 11t^2 + 374t$$

$$a(t) = v'(t) = 22t + 374$$ ✓

b) What is acc. when velocity = 9537 ft/s.

$$9537 = 11t^2 + 374t$$

$$0 = 11t^2 + 374t - 9537$$

$$0 = t^2 + 34t - 867$$

$$t = \frac{-34 \pm \sqrt{34^2 - 4(1)(-867)}}{2}$$

$$= \frac{-34 \pm \sqrt{1156 + 3468}}{2}$$

$$= \frac{-34 \pm \sqrt{4624}}{2}$$

$$= \frac{-34 \pm 68}{2} = \frac{-102}{2} \text{ or } \frac{34}{2}$$

$$= \cancel{-51\,s} \text{ or } \boxed{17\,s}$$

$$a(17) = 22(17) + 374 = 748 \text{ ft/s}^2.$$

Lesson 13: Implicit differentiation

Explicit functions: $y = f(x)$, $y = 3\sin(x)$

$$g(x) = 10x + x^{3/4}$$

Implicit function: $f(x, y) = 0$

Examples

① $y - 3x = 0$

② $3y^2 = 1 + 5x^2$

③ $\sin(x + 3y) = 2xy$

④ $e^{xy} = 2x$

① $y - 3x = 0$ find dy/dx.

$$\frac{d}{dx}[y - 3x] = \frac{d}{dx}[0]$$

$$\frac{d}{dx}(y) + \frac{d}{dx}(-3x) = 0$$

$$\frac{dy}{dx} - 3 = 0$$

$$\frac{dy}{dx} = 3 \checkmark$$

$y = 3x$

$$\frac{dy}{dx} = 3 \checkmark$$

(2) find the equation of tan. line $2x^4 = 4y^2 + 6x^2$ at $(2, \sqrt{2})$.

$y - y_1 = m(x - x_1)$

m: slope = derivative $\quad m = \left.\frac{dy}{dx}\right|_{(2,\sqrt{2})}$

(x_1, y_1): point on tan. line $(2, \sqrt{2})$

$$\frac{d}{dx}\left[2x^4\right] = \frac{d}{dx}\left[4y^2 + 6x^2\right]$$

$$8x^3 = \frac{d}{dx}\left[4y^2\right] + \frac{d}{dx}\left[6x^2\right]$$

$$8x^3 = 4 \cdot 2(y)^1 \cdot \frac{dy}{dx} + 12x$$

out: $g(u) = 4u^2$

in: y

$$8x^3 - 12x = 8y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{8x^3 - 12x}{8y}$$

$$\left.\frac{dy}{dx}\right|_{(2,\sqrt{2})} = \frac{8 \cdot 2^3 - 12 \cdot 2}{8\sqrt{2}} = \frac{64 - 24}{8\sqrt{2}} = \frac{40}{8\sqrt{2}} = \frac{5}{\sqrt{2}}$$

"x" "y"

$$y - \sqrt{2} = \frac{5}{\sqrt{2}}(x - 2)$$

✓

(3) $\sin(x+3y) = 2xy$ find dy/dx

$$\frac{d}{dx}\left[\sin(x+3y)\right] = \frac{d}{dx}\left[2xy\right]$$

$$\cos(x+3y)\cdot\frac{d}{dx}\left[x+3y\right] = 2\cdot y + 2x\cdot\frac{dy}{dx}$$

$$\cos(x+3y)\left(1+3\frac{dy}{dx}\right) = 2y + 2x\frac{dy}{dx}$$

$$\frac{d}{dy}\left[y\right] = 1 \qquad \frac{d}{dx}(y) = \frac{d}{dx}(f(x)) = f'(x) \neq 1$$

$$\cos(x+3y) + 3\cos(x+3y)\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$\cos(x+3y) - 2y = -3\cos(x+3y)\frac{dy}{dx} + 2x\frac{dy}{dx}$$

$$\cos(x+3y) - 2y = \left(-3\cos(x+3y) + 2x\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\cos(x+3y) - 2y}{-3\cos(x+3y) + 2x}$$

④ $\tan(x/y) = 10x \qquad dy/dx\ ?$

$$\frac{d}{dx}\left[\tan(x/y)\right] = \frac{d}{dx}\left[10x\right]$$

$$\sec^2\left(\frac{x}{y}\right) \cdot \frac{d}{dx}\left[\frac{x}{y}\right] = 10$$

$$\frac{d}{dx}\left[\frac{x}{y}\right] = \frac{1 \cdot y - x \cdot dy/dx}{y^2}$$

$$\sec^2\left(\frac{x}{y}\right) \cdot \frac{y - x\, dy/dx}{y^2} = 10$$

$$\frac{y - x\, dy/dx}{y^2} = \frac{10}{\sec^2(\frac{x}{y})} = 10\cos^2\left(\frac{x}{y}\right)$$

$$y - x\frac{dy}{dx} = 10y^2\cos^2\left(\frac{x}{y}\right)$$

$$-x\frac{dy}{dx} = -y + 10y^2\cos^2\left(\frac{x}{y}\right)$$

$$\frac{dy}{dx} = \frac{y - 10y^2\cos^2(x/y)}{x} \quad \checkmark$$