Lesson 14: Related rates

$$y^2 + 2x = 3 \qquad \text{find } dy/dx$$

$$\frac{d}{dx}\left[y^2 + 2x\right] = \frac{d}{dx}(3)$$

$$2y\frac{dy}{dx} + 2 = 0$$

① Suppose that x and y are functions of t ($x = x(t)$ and $y = y(t)$)

$$7x^9 y = 14 \qquad dx/dt = 2. \quad \text{find } dy/dt \text{ at } x = 1$$

$$\frac{d}{dt}\left(7x^9 y\right) = \frac{d}{dt}\left(14\right) = 0$$

$$\frac{d}{dt}(7x^9)\cdot y + 7x^9 \cdot \frac{d}{dt}(y) = 0$$

$$63x^8 \cdot \frac{dx}{dt} \cdot y + 7x^9 \cdot \frac{dy}{dt} = 0$$

$$63(1)^8 \cdot 2 \cdot y + 7(1)^9 \cdot \frac{dy}{dt} = 0$$

$$126y + 7\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-126 \cdot 2}{7}$$

$$7x^9 y = 14$$

$$7(1)^9 y = 14$$

$$y = 2$$

(2) The radius of a circle is increasing at a rate of 9 cm/min. Find $\underbrace{\text{R.o.C. of circumference}}_{\text{derivative of } C}$ with respect to time when

when $r = 5$ cm (radius).

$C = 2\pi r$

$\frac{dr}{dt}$ = "R.o.C. of radius wrt. time"

= 9 cm/min

$$\frac{dC}{dt} = \frac{d}{dt}(2\pi r) = 2\pi \frac{dr}{dt} = 2\pi \cdot 9 = 18\pi \text{ cm/min}$$

(3) All edges of ^cube are shrinking at a rate of 35 cm/s

How fast is the surface area decreasing when

R.O.C = derivative

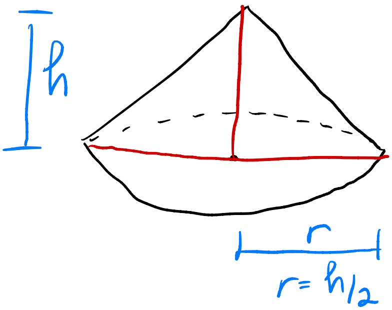
each edge is 11 cm.

$S = 6a^2$ $\quad$ a = length of edges

$$\frac{dS}{dt} = \frac{d}{dt}(6a^2) = 12a \cdot \frac{da}{dt} = 12(11)(35) = 4620 \text{ cm/s}$$

4. Sand is being poured on a pile at a rate of $11\,cm^3/s$. $\frac{dV}{dt} = 11$

length of diameter of the base = altitude.



How fast is the alt. changing when the pile is 3 cm high? $dh/dt = ?$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$$

Lesson 14: Related rates.

1. Suppose x and y are functions of t
   ($x = g(t)$ and $y = f(t)$).

   $7x^9 y = 14$, $dx/dt = 2$. find dy/dt at $\underline{x = 1}$.

$$\frac{d}{dt}[7x^9 y] = \frac{d}{dt}[14] = 0$$

$$\frac{d}{dt}[7x^9]\cdot y + 7x^9 \cdot \frac{d}{dt}[y] = 0$$

$$63 \cdot x^8 \cdot \frac{dx}{dt} \cdot y + 7x^9 \cdot \frac{dy}{dt} = 0$$

$$63(1)^8 \cdot 2 \cdot y + 7(1)^9 \cdot \frac{dy}{dt} = 0$$

$$126y + 7\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{126 \cdot 2}{7} = -36 \checkmark$$

$$7x^9 y = 14$$

$$7(1)^9 y = 14$$

$$y = 2$$

2. The radius is increasing at a rate of 9 cm/min.
   Find $\underbrace{\text{R.o.C. of circumference}}$ of circle $\underbrace{\text{w.r.t. time}}$
   when $r = 5$ cm. find derivative of C w.r.t. to t

$C = 2\pi r$ C: circumference
r: radius

$$\frac{dC}{dt} = \frac{d}{dt}[2\pi r]$$

$$= 2\pi \frac{dr}{dt}$$

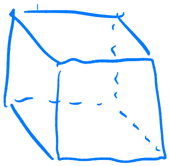
$$= 18\pi$$

$\frac{dr}{dt}$ = "R.o.C. of radius wrt. time"
= 9 cm/min

$$\left.\frac{dC}{dt}\right|_{r=5} = 18\pi \text{ cm/min.} \checkmark$$

(3) all the edges of a cube are shrinking at a rate of 35 cm / sec.

How fast is surface area of the cube decreasing w.r.t. to time when edge length is 11 cm?



A: surface area $\quad A = 6s^2$

s: length of edge.

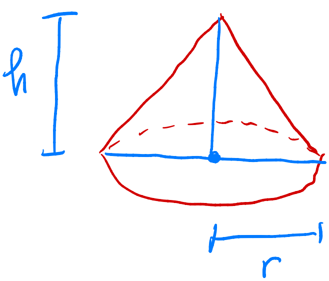
$\frac{ds}{dt} = 35 \text{ cm/sec} \qquad \left.\frac{dA}{dt}\right|_{s=11} = ?$

$$\frac{dA}{dt} = \frac{d}{dt}(6s^2) = 12(s)' \cdot \frac{ds}{dt} = 12s \cdot 35 = 420s$$

$$\left.\frac{dA}{dt}\right|_{s=11} = 420 \cdot 11 = 4620 \text{ cm}^2/\text{sec} \quad ✓$$

(4) • Sand is being poured into a pile at a rate of $11 \text{ cm}^3/\text{sec}$.

• length of the diameter of base = altitude.



How fast is alt. changing w.r.t time when pile is 3cm tall.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

$\frac{dV}{dt} = 11 \text{ cm}^3/\text{sec} \qquad \left.\frac{dh}{dt}\right|_{h=5} = ?$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{\pi}{12} \cdot h^3\right) = \frac{\pi}{12} \cdot 3\left(h \right)^2 \cdot \frac{dh}{dt}$$

$$11 = \frac{\pi}{4} \cdot h^2 \cdot \frac{dh}{dt}$$

$$11 = \frac{\pi}{4} \cdot 9 \cdot \frac{dh}{dt} \qquad \left.\frac{dh}{dt}\right|_{h=3} = \frac{11 \cdot 4}{9\pi} = \frac{44}{9\pi} \text{ cm/sec.}$$