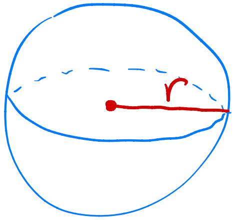
HW 14 # 4



radius r decreasing at a rate of 1.2 cm/sec

1.) How fast is volume dec. when $r = 10$cm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}(r^3)$$

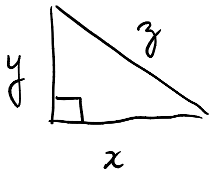
$$= \frac{4}{3}\pi \cdot 3(r)^2 \cdot \frac{dr}{dt}$$

$$= \frac{4}{3}\pi \cdot 3(10)^2 \cdot 1.2$$

$$= 4\pi \cdot 10 \cdot 12$$

$$= 480\pi \text{ cm}^3/\text{sec}$$

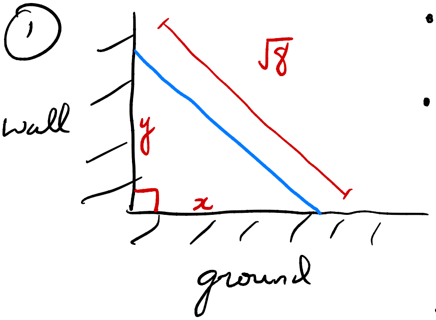
Lesson 15: Related rates.



$$x^2 + y^2 = z^2$$

Pythagorean Thm

(1)



- length of ladder is $\sqrt{8}$ m
- The base of ladder is moving away from wall at a rate of $\frac{3}{10}$ m/s
- How fast is the head of ladder moving down when the base is 2m away from the wall?

$$x^2 + y^2 = (\sqrt{8})^2 = 8$$

$$\frac{dx}{dt} = \frac{3}{10} \text{ m/s}$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}[8]$$

$$\left.\frac{dy}{dt}\right|_{\text{2m away from wall}} = ?$$

$$2(x)\cdot\frac{dx}{dt} + 2(y)\cdot\frac{dy}{dt} = 0$$

$$2x \cdot \frac{3}{10} + 2y\frac{dy}{dt} = 0$$

When we are 2m away from wall

$$x = 2$$

$$(2)^2 + y^2 = 8$$

$$y^2 = 4$$

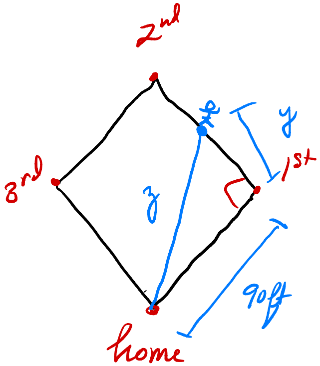
$$y = 2$$

$$\frac{dy}{dt} = \frac{-2x\cdot\frac{3}{10}}{2y}$$

$$= \frac{-2\cdot 2\cdot 3/10}{2\cdot 2} = \boxed{-\frac{3}{10} \text{ m/s}}$$

head of ladder is moving down at a rate of 3/10 m/s.

(2)



- This square w/ side length 90ft
- player is running at a rate of 12ft/sec
- At what rate is the players dist. from home inc. when they are half b/w 1st and 2nd.

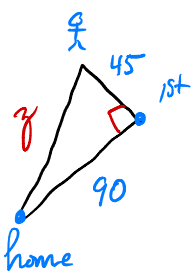
$$(90)^2 + y^2 = z^2 \qquad \frac{dy}{dt} = 12 \text{ ft/sec} \qquad \frac{dz}{dt} = ?$$

$$\frac{d}{dt}\left[90^2 + y^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$0 + 2(y)\cdot\frac{dy}{dt} = 2(z)\cdot\frac{dz}{dt}$$

$$2y \cdot 12 = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{2\cdot 12y}{2z} = \frac{12y}{z} \quad \checkmark$$

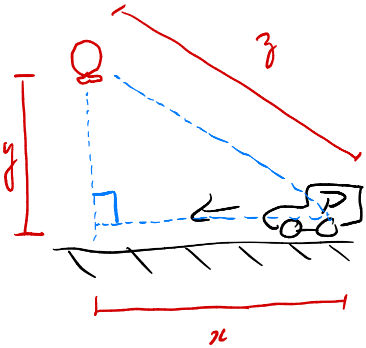


$$z^2 = (90)^2 + (45)^2 \qquad \frac{dz}{dt} = \frac{12\cdot 45}{\sqrt{1025}} \text{ ft/sec}$$

$$z^2 = 10125$$

$$z = \sqrt{10125}$$

(3) • A Balloon at a height of 50m is rising at a rate of 5 m/sec.



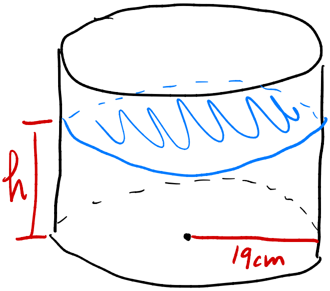
• The Boiler maker special driving towards Balloon at a speed of 10 m/s.

• How fast is dist. B/w BMS. and the balloon. changing 10 seconds latter.

$$x^2 + y^2 = z^2 \qquad \frac{dy}{dt} = 5, \quad \frac{dx}{dt} = 10, \quad \frac{dz}{dt} = ?$$

See last page for soln.

HW 14: #5



- Water is being drained at a rate of $15\ cm^3/sec$
- How fast is the height of water dec?

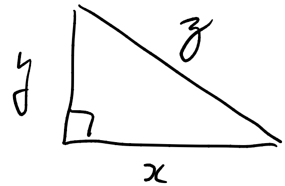
$$V = \pi r^2 h = 19^2 \pi h \qquad \frac{dV}{dt} = -15, \quad \frac{dh}{dt} = ?$$

$$\frac{dV}{dt} = 19^2 \pi \frac{d}{dt}(h) = 19^2 \pi \frac{dh}{dt}$$

$$-15 = 19^2 \pi \cdot \frac{dh}{dt} \qquad \frac{dh}{dt} = \frac{-15}{19^2 \pi}\ cm/sec$$

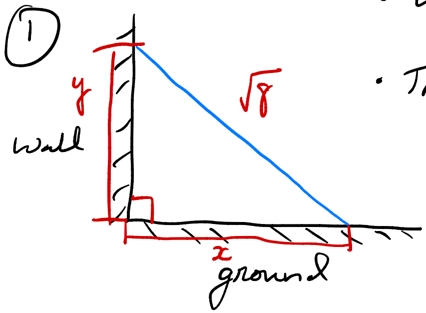
speed the height dec at is $\frac{15}{19^2 \pi}\ cm/sec$

Lesson 15: Related rates



Pythagorean Thm:

$$x^2 + y^2 = z^2$$



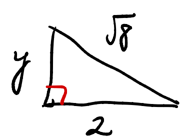
- We have a ladder of length $\sqrt{8}$ m
- The base of the ladder is pulled away from wall at a rate of $3/10$ m/s
- How fast is the head of ladder moving down when the base is 2m from the wall?

$$x^2 + y^2 = (\sqrt{8})^2 = 8 \, , \quad \frac{dx}{dt} = \frac{3}{10} \text{ m/s}, \quad \frac{dy}{dt} = ??$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}[8]$$

$$2(x) \cdot \frac{dx}{dt} + 2(y) \cdot \frac{dy}{dt} = 0$$

$$2x \cdot \frac{3}{10} + 2y \cdot \frac{dy}{dt} = 0$$



$$\frac{dy}{dt} = \frac{-3/10\, x}{y}$$

$$= \frac{-3/10 \cdot 2}{2} = -\frac{3}{10}$$

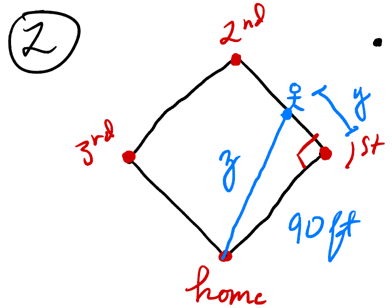
$$2^2 + y^2 = 8$$

$$y^2 = 4$$

$$y = 2$$

Speed at which the head of the ladder falls is $3/10$ m/s

(2)



- This a square of side length 90 ft.
- player is running at a rate of 12 ft/s.
- At what rate is the dist. b/w the player and home encreasing when the player is half b/w 1st and 2nd?

$$90^2 + y^2 = z^2 , \quad \frac{dy}{dt} = 12 \text{ ft/s} , \quad \frac{dz}{dt} = ??$$

$$\frac{d}{dt}\left[90^2 + y^2 \right] = \frac{d}{dt}\left[z^2 \right]$$

$$0 + 2(y) \cdot \frac{dy}{dt} = 2(z) \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{12y}{z}$$

$$y = 45$$

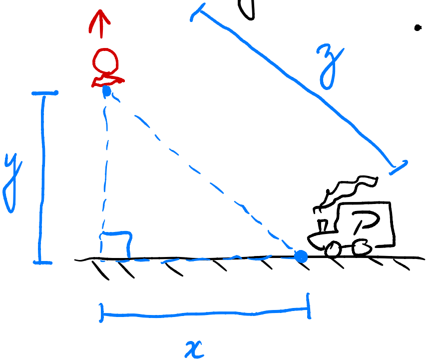
$$z^2 = 90^2 + 45^2$$

$$= 10125$$

$$z = \sqrt{10125}$$

$$= \frac{12 \cdot 45}{\sqrt{10125}} \checkmark$$

③ • A Balloon is at a height of 50 m and rising at a rate of 5 m/s.



• The Boiler maker special is driving towards balloon at a rate of 10 m/s.

• How fast is the dist. B/w the two inc. 10 seconds later?

* Note at time $t=0$ the Balloon is 50m above the ground and the train is directly below the balloon.

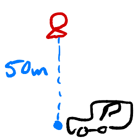
$$x^2 + y^2 = z^2 \quad , \quad \frac{dy}{dt} = 5 \text{ m/s} \, , \quad \frac{dx}{dt} = 10 \text{ m/s} \quad , \quad \frac{dz}{dt} = ??$$

for ease of calculations assume positive

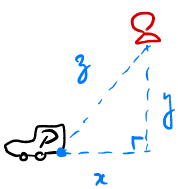
$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

at $t = 0$ sec



at $t = 10$ sec



Since the Balloon is rising at a rate of 5 m/s, then 10 sec. later the height of balloon $y = 50 + 5(10) = 100$ m.

Similarly since train has speed 10 m/s, $x = 10(10) = 100$ m

Thus $z = \sqrt{x^2 + y^2} = 100\sqrt{2}$ m

Thus $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

will simplify to

$$2(100)\cdot(10) + 2(100)\cdot(5) = 2\cdot 100\sqrt{2}\,\frac{dz}{dt}$$

$$\boxed{\therefore \quad \frac{dz}{dt} = \frac{2000+1000}{200\sqrt{2}} = \frac{15}{\sqrt{2}} \ \text{m/s}}$$