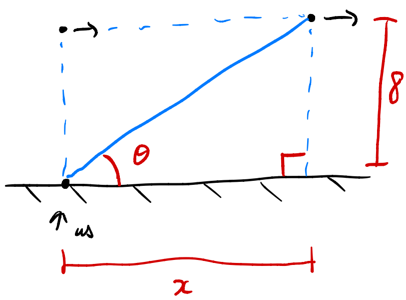
HW 15 # 7



plane traveling at 350 mph
alt. of 8 m.

$$\tan\theta = \frac{8}{x} \qquad \frac{dx}{dt} = 350, \quad \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = ?$$

$$\frac{d}{dt}[\tan\theta] = \frac{d}{dt}\left[\frac{8}{x}\right] = \frac{d}{dt}\left[8x^{-1}\right]$$

$$\sec^2(\theta)\cdot\frac{d\theta}{dt} = -8x^{-2}\cdot\frac{dx}{dt}$$

$$\tan\frac{\pi}{6} = \frac{8}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{8}{x}$$

$$x = 8\sqrt{3}$$

$$\sec^2\left(\frac{\pi}{6}\right)\cdot\frac{d\theta}{dt} = -8\left(8\sqrt{3}\right)^{-2}\cdot 350$$

$$\frac{1}{3/4}\cdot\frac{d\theta}{dt} = \frac{-8}{64\cdot 3}\cdot 350$$

$$\frac{1}{\cos^2(\pi/6)} = \frac{1}{(\sqrt{3}/2)^2}$$

$$\frac{d\theta}{dt} = \frac{\cancel{3}}{4}\cdot\frac{-1}{8\cdot\cancel{3}}\cdot 350 = \frac{-175}{16}$$

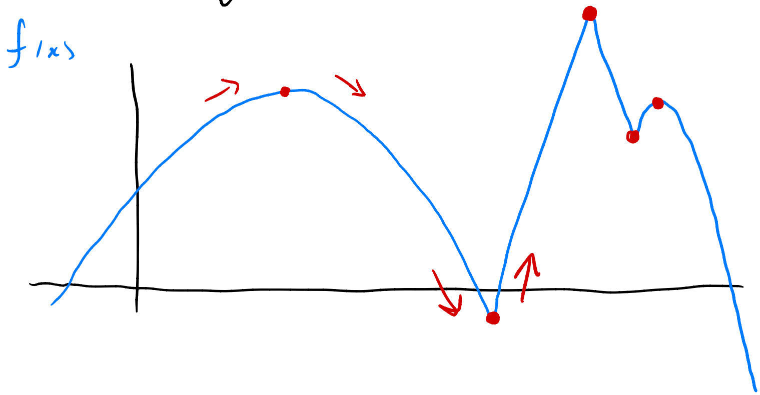
angle of elevation is decr. at $\frac{175}{16}$ rad/hour.

Lesson 16: Relative extrema and critical numbers.

Relative extrema

function goes up ↗ and then down ↘ relative maxima

(or goes down ↘ and then up ↗) reltive minimum



a critical point of a function $f(x)$ is a point $x=c$ such that either

$$1)\ f'(c)=0 \quad \text{or} \quad 2)\ f'(c) = \text{DNE}$$

How to find relative extrema

Step 1: find all critical points could have false positives!

Step 2: determine which critical pts are relative extrema by checking derivative.

Examples   find critical pts

① $y = 3x^2 - 6x$

$y' = 6x - 6$ $y' = 0$ or $y' = \text{DNE}$

y' always exists

$0 = 6x - 6$

$x = 1$

② $y = 9x^2 - 9/x^2$

$y' = 18x + 18/x^3$

$y' = 0$

$18x + \frac{18}{x^3} = 0$

$\frac{18x^4 + 18}{x^3} = 0$

$18x^4 + 18 = 0$

$x^4 + 1 = 0$

$x^4 = -1$ no solution!

$y' = \text{DNE}$

$y' = \frac{18x^4 + 18}{x^3}$

$x^3 = 0$

$x = 0$

y is not defined at $x = 0$ so throw this point out!

no critical pts!

③ $y = \dfrac{4x^2+8}{7x}$

$$y' = \frac{(8x)(7x) - (4x^2+8)(7)}{(7x)^2}$$

$$= \frac{56x^2 - 28x^2 - 56}{49x^2}$$

$$= \frac{28x^2 - 56}{49x^2}$$

$y' = 0$ when numerator $= 0$

$$28x^2 - 56 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

$y' = \text{DNE}$ denominator $= 0$

$$49x^2 = 0$$
$$x = 0$$

y is not defined here.

(4) $y = 2\cos(8x) + 8x$ critical pts on the interval $0 \le x \le \pi$

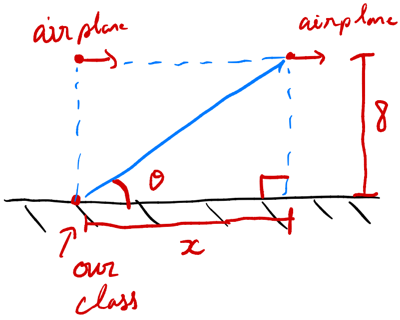
$y' = -16\sin(8x) + 8$

$y' = 0$: $16\sin(8x) = 8$

$\sin(8x) = 1/2$

See last page for soln.

HW 15 #7



plane is flying at 350 mph at an alt. of 8 miles.

Find the rate at which the angle of elevation decreases when $\theta = \frac{\pi}{6}$

$$\frac{dx}{dt} = 350, \quad \left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}} = ??$$

$$\tan\theta = \frac{8}{x}$$

$$\frac{d}{dt}\left[\tan\theta\right] = \frac{d}{dt}\left[\frac{8}{x}\right] = \frac{d}{dt}\left[8x^{-1}\right]$$

$$\sec^2(\theta)\cdot\frac{d\theta}{dt} = -8x^{-2}\cdot\frac{dx}{dt}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{8}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{8}{x}$$

$$x = 8\sqrt{3}$$

$$\sec^2\left(\frac{\pi}{6}\right)\cdot\frac{d\theta}{dt} = -8(8\sqrt{3})^{-2}\cdot 350$$

$$\frac{1}{3/4}\cdot\frac{d\theta}{dt} = \frac{-8}{8^2\cdot 3}\cdot 350$$

$$\frac{d\theta}{dt} = \frac{-8\cdot 3}{8^2\cdot 3\cdot 4}\cdot 350$$

$$= -\frac{175}{16}\ \text{rad/hr}$$

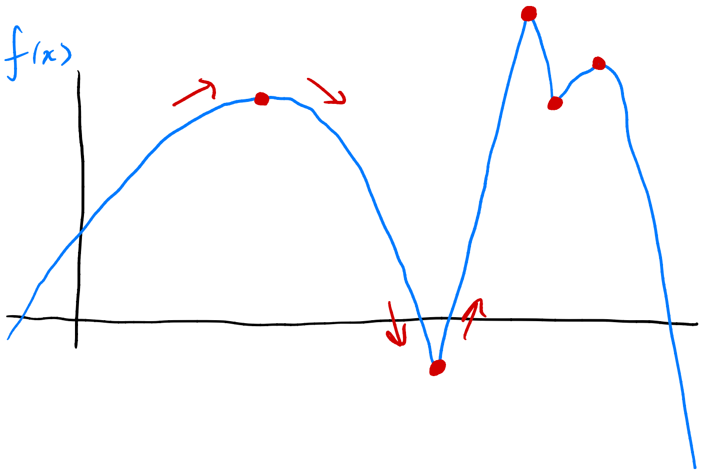
angle of elevation is decr. at a rate of $175/16$ rad/hr.

Lesson 16: Relative extrema and critical numbers.

Relative extrema

function goes up ↗ and then down ↘ → relative max

or goes down ↘ and then up ↗ → relative min



A critical point of a function $f(x)$ is a point at $x = c$ such that either

1) $f'(c) = 0$ or 2) $f'(c) = DNE$

How to find relative extrema

Step 1: find all critical points. — We may have false positives

Step 2: look at the derivatives near the critical points.

Examples: find critical points

(1) $y = 3x^2 - 6x$

$y' = 6x - 6$

$y' = 0$ or $y' = DNE$

$6x - 6 = 0$ $\quad$ y' always exists ✗

$6x = 6$

$\underline{x = 1}$ ✓

(2) $y = 9x^2 - 9/x^2$

$y' = 18x + 18/x^3$

$y' = 0$ or $y' = DNE$

$$18x + \frac{18}{x^3} = 0$$

$$\frac{18x^4 + 18}{x^3} = 0 \quad \longrightarrow \quad \frac{18x^4 + 18}{x^3} = DNE$$

when $x^3 = 0$

$x = 0$

$18x^4 + 18 = 0$

$x^4 = -1$

no soln!

lar aapn $\Big\{$ y is not defined at $x = 0$
its an asymptote
so disregard this point.

(3) $y = \dfrac{4x^2+8}{7x}$

$$y' = \frac{(8x)(7x) - (4x^2+8)(7)}{(7x)^2}$$

$$= \frac{56x^2 - 28x^2 - 56}{49x^2}$$

$$= \frac{28x^2 - 56}{49x^2}$$

$y' = 0$

when numerator $= 0$

$$28x^2 - 56 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

$y' = \text{DNE}$

when denominator $= 0$

$$49x^2 = 0$$
$$x = 0$$

disregard $x = 0$ since y has an asymptote there.

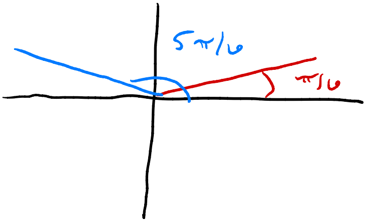
(4) $y = 2\cos(8x) + 8x$ find critical points on $0 \leq x \leq \pi$

$$y' = -2\sin(8x) \cdot 8 + 8$$
$$= -16\sin(8x) + 8$$

$y' = 0$: $-16\sin(8x) + 8 = 0$

$$16\sin(8x) = 8$$
$$\sin(8x) = 1/2$$

zero times around circle

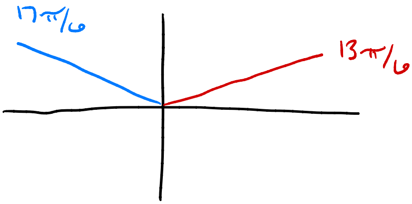


$$8x = \frac{\pi}{6} \qquad 8x = \frac{5\pi}{6}$$

$$x = \frac{\pi}{6\cdot 8} \qquad x = \frac{5\pi}{6\cdot 8}$$

$$= \frac{\pi}{48} \qquad = \frac{5\pi}{48}$$

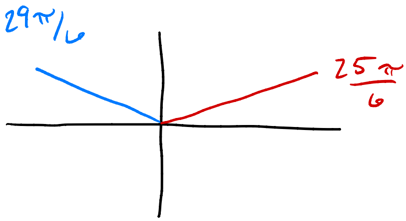
1 times around



$$8x = \frac{13\pi}{6} \qquad 8x = \frac{17\pi}{6}$$

$$x = \frac{13\pi}{48} \qquad x = \frac{17\pi}{48}$$

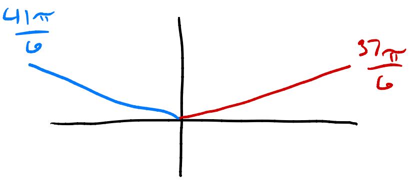
2 times around



$$8x = \frac{25\pi}{6} \qquad 8x = \frac{29\pi}{6}$$

$$x = \frac{25\pi}{48} \qquad x = \frac{29\pi}{48}$$

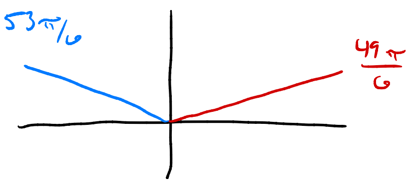
3 times around



$$8x = \frac{37\pi}{6} \qquad 8x = \frac{41\pi}{6}$$

$$x = \frac{37\pi}{48} \qquad x = \frac{41\pi}{48}$$

4 times around



$$8x = \frac{49\pi}{6} \qquad 8x = \frac{53\pi}{6}$$

$$x = \frac{49\pi}{48} > \pi \qquad x = \frac{53\pi}{48} > \pi$$

too large!

Critical points: $\frac{\pi}{48}, \frac{5\pi}{48}, \frac{13\pi}{48}, \frac{17\pi}{48}, \frac{25\pi}{48}, \frac{29\pi}{48}, \frac{37\pi}{48}, \frac{41\pi}{48}$.