HW 16 #11 $y = 2\cos(8x) + 8x$ critical numbers on the interval $(0, \pi)$.

$$y' = -2\sin(8x) \cdot 8 + 8$$

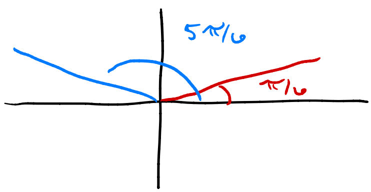
$$= -16\sin(8x) + 8$$

$y' = 0$ or y' DNE

never happens.

$16\sin(8x) = 8$ solve for x when $0 \leq x \leq \pi$

$$\sin(8x) = 1/2$$



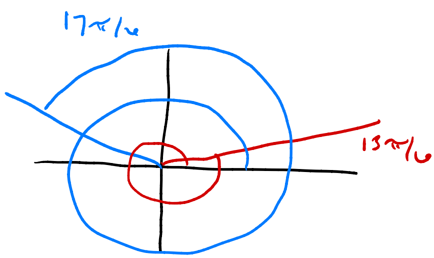
$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{5\pi}{6}\right) = 1/2$

$\rightsquigarrow$

$8x = \frac{\pi}{6}$ $x = \frac{\pi}{48}$

$8x = \frac{5\pi}{6}$ $x = \frac{5\pi}{48}$



$8x = \frac{13\pi}{6}$ $x = \frac{13\pi}{48}$

$8x = \frac{17\pi}{6}$ $x = \frac{17\pi}{48}$

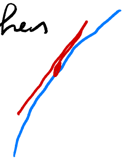
We need to go around 3 times.

Lesson 17: Increasing and decreasing functions; First derivative test

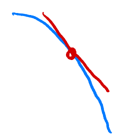
Recall an interval (a, b) = all points x such that $a < x < b$

Increasing / decreasing functions

- If $f'(x) > 0$ for all x in (a, b), then f is increasing on (a, b).



- If $f'(x) < 0$ for all x in (a, b), then f is decreasing on (a, b).



First derivative test

A way finding relative extrema.

1) find all critical points of f
2) See how the derivative changes near the critical points.

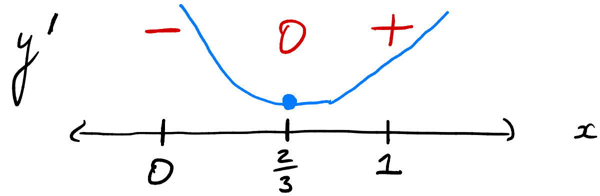
Examples

① $y = 6x^2 - 8x$ find the relative extrema.

$y' = 12x - 8$ $\qquad$ $y' = 0$ $\qquad$ or $\qquad$ y' DNE never happens.

$$0 = 12x - 8$$
$$12x = 8$$
$$x = 8/12 = 2/3$$

Critical numbers : $x = 2/3$



$y'(1) = 12(1) - 8 = 12 - 8 = 4$

$y'(0) = 12(0) - 8 = -8$

$x = \frac{2}{3}$ is a relative minimum of y.

(2) $y = 5 + 6x - 8x^3$ find relative extrema.

$y' = 0 + 6 - 24x^2$

$y' = 0$ or y' DNE
never happens.

$6 - 24x^2 = 0$

$24x^2 = 6$

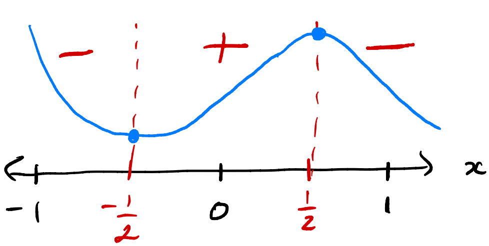
$x^2 = 6/24$

$x^2 = 1/4$

$x = \pm 1/2$

Critical numbers : $x = \frac{1}{2}$ $x = -\frac{1}{2}$

$y' = 6 - 24x^2$



$-\frac{1}{2}$ is relative minimum

$\frac{1}{2}$ is a relative maximum.

$$y'(-1) = 6 - 24(-1)^2 = 6 - 24 = -18$$

So we know $(-\infty, -\frac{1}{2})$ y is decreasing

$$y'(0) = 6 - 24(0)^2 = 6$$

y is increasing on $(-\frac{1}{2}, \frac{1}{2})$

$y'(1) = 6 - 24(1)^2 = -18$ y is decreasing on $(\frac{1}{2}, \infty)$

③ Let $f(x)$ be a function w/ first derivative given by $f'(x) = (x-3)^2(x+7)$. Find all relative extrema.

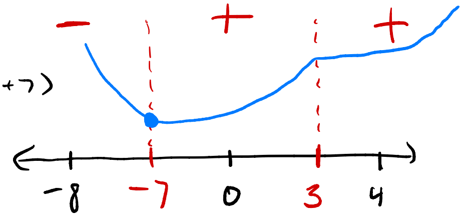
$f'(x) = 0$ or $f'(x)$ DNE

never happens.

$0 = (x-3)^2(x+7)$

$x = 3$ or $x = -7$ Critical numbers: $x = 3, -7$

$f'(x) = (x-3)^2(x+7)$



−7 relative minimum.

3 is false positive.

$$f'(-8) = (-8-3)^2(-8+7)$$
$$= (-11)^2(-1)$$
$$= 121\,(-1)$$
$$= -121$$

f is decreasing on $(-\infty, -7)$.

$$f'(0) = (0-3)^2(0+7) = 9 \cdot 7 = 63$$

f is inc. on $(-7, 3)$

$$f'(4) = (4-3)^2(4+7) = 1^2 \cdot (11) = 11$$

f is inc. on $(3, \infty)$

HW 16 # 11 $y = 2\cos(8x) + 8x$ find critical numbers on $(0, \pi)$.

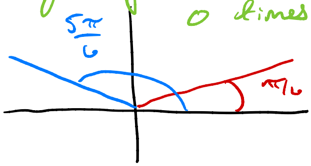
$y' = -16\sin(8x) + 8$

$y' = 0$ or y' DNE never happens

$16\sin(8x) = 8$

$\sin(8x) = \frac{1}{2}$

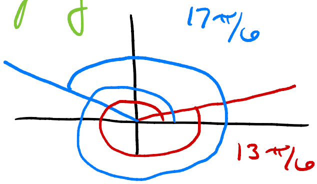
going around 0 times



$\sin(\frac{\pi}{6}) = \frac{1}{2} \rightsquigarrow 8x = \frac{\pi}{6} \quad x = \frac{\pi}{48}$

$\sin(\frac{5\pi}{6}) = \frac{1}{2} \quad 8x = \frac{5\pi}{6} \quad x = \frac{5\pi}{48}$

going around 1 time



$8x = \frac{13\pi}{6} \quad x = \frac{13\pi}{48}$

$8x = \frac{17\pi}{6} \quad x = \frac{17\pi}{48}$

We will need to go around 3 times to get all of critical numbers

Lesson 17: Increasing and decreasing functions; First derivative test

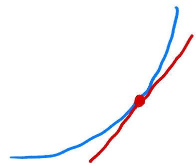
<u>Recall</u> an interval (a, b) = all points x such that

$$a < x < b$$

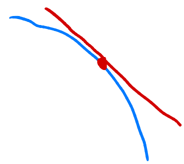
Note that a may be $-\infty$
b may be $+\infty$

Increasing / decreasing functions

- If $f'(x) > 0$ on (a, b), then f is increasing on (a, b).



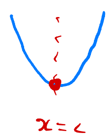
- If $f'(x) < 0$ on (a, b), then f is decreasing on (a, b)



Recall that $x = c$ (critical number) is a

- relative maximum of f if the derivative of f switches from positive to negative

- relative minimum of f if the derivative of f switches from negative to positive



First derivative test

1) find all critical numbers of f

2) Use f' to see how the slope changes near the critical points.

Examples

① $y = 6x^2 - 8x$ find all relative extrema.

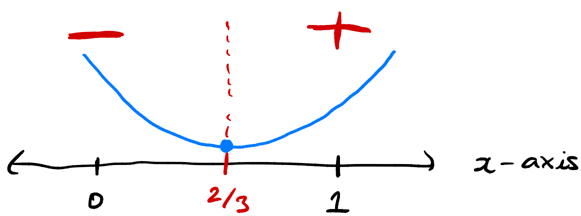
$y' = 12x - 8$

$y' = 0$ or y' DNE never happens

$12x - 8 = 0$

$12x = 8$

$x = 2/3$ Critical numbers : $x = 2/3$.



$y'(0) = 12(0) - 8 = -8$ y is decreasing on $(-\infty, \frac{2}{3})$

$y'(1) = 12(1) - 8 = 4$ y is increasing on $(\frac{2}{3}, \infty)$

So $x = 2/3$ is a relative minimum of y.

② $y = 5 + 6x - 8x^3$ find relative extrema

$y' = 0 + 6 - 24x^2$

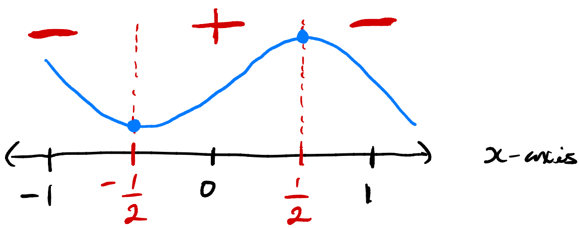
$y' = 0$ or y' DNE never happens

$6 - 24x^2 = 0$

$24x^2 = 6$

$x^2 = 1/4$

$x = \pm 1/2$ Critical points: $x = \frac{1}{2}, -\frac{1}{2}$



$y'(-1) = 6 - 24(-1)^2 = 6 - 24 = -18$

y is decreasing on $(-\infty, -1/2)$

$y'(0) = 6 - 24(0)^2 = 6$ y is inc. on $(-\frac{1}{2}, \frac{1}{2})$

$y'(1) = 6 - 24(1)^2 = -18$ y is decr. on $(\frac{1}{2}, \infty)$

$x = -\frac{1}{2}$ is relative minimum of y

$x = \frac{1}{2}$ is a relative maximum of y.

③ Let $f(x)$ be a function with a first derivative given by $f'(x) = (x-3)^2(x+7)$.

Find all relative extrema of f.

$f'(x) = 0$ or $f'(x)$ DNE

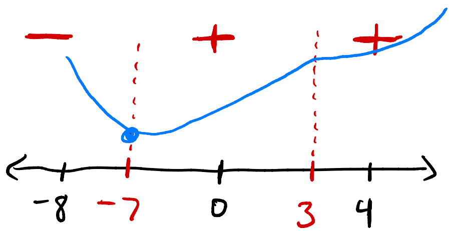
never happens.

$0 = (x-3)^2(x+7)$

$x = 3$, $x = -7$

Critical numbers : $x = 3, -7$

$x=-7$ is a relative minimum



$x=3$ is not a relative extrema

false positive.

$$f'(-8) = (-8-3)^2(-8+7) = (-11)^2(-1) = 121(-1) = -121$$

f is dec. on $(-\infty, -7)$

$$f'(0) = (0-3)^2(0+7) = 9 \cdot 7 = 63$$

f is inc. on $(-7, 3)$

$$f'(4) = (4-3)^2(4+7) = (1)^2(11) = 11$$

f is inc. on $(3, \infty)$.