HW 18 #5 | $f(x) = \frac{x^5}{10} + \frac{x^4}{8} + 1$

$f'(x) = \frac{1}{2}x^4 + \frac{1}{2}x^3$

$f''(x) = 2x^3 + \frac{3}{2}x^2$

$f''(x) = 0$ or $f''(x) = DNE$ never happens

$0 = 2x^3 + \frac{3}{2}x^2$

$= x^2\left(2x + \frac{3}{2}\right)$

$x = 0$ or $2x + \frac{3}{2} = 0$

$2x = -\frac{3}{2}$

$x = -3/4$

potential inflection pts. 0 and $-3/4$

$f''(-1) = (-1)^2(2(-1) + 3/2) < 0$

$f''(-1/2) = (-1/2)^2(2(-1/2) + 3/2) > 0$

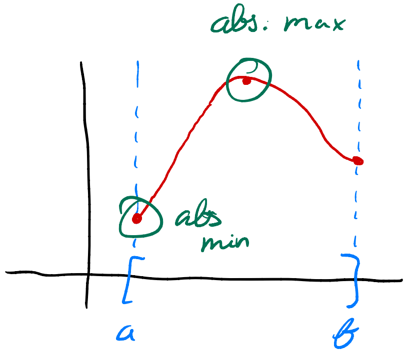
$f''(1) = (1)^2(2(1) + 3/2) > 0$

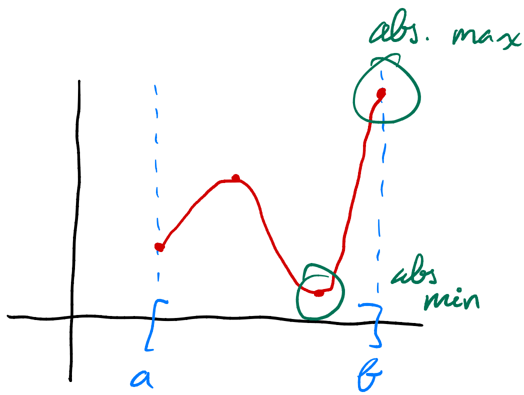
Lesson 19: Absolute extrema on an interval

Let $f(x)$ be a function on interval I

1) $x = c$ is an absolute max if $f(c)$ is the largest value of f on I

2) $x = c$ is an absolute min if $f(c)$ is the smallest value of f on I





How to find abs. ext. on $[a, b]$

1) find critical numbers

2) Evaluate f at critical numbers and a, b

3) Determine largest / smallest value.

Examples

(1) $y = \underbrace{5x}\underbrace{e^{-x}} + 2$ on $[0,3]$ find abs. ext.

$y' = 5e^{-x} + 5x(-e^{-x}) + 0$

$= 5e^{-x}(1-x)$

Critical numbers : $y' = 0$ or y' DNE
never happens

$0 = 5e^{-x}(1-x)$

e^{-x} is always positive and never equal to 0.

$0 = 1 - x$

$x = 1$ ← only critical number.

x	$f(x) = 5xe^{-x} + 2$
0	$f(0) = 2$ ← abs. min
1	$f(1) = 5e^{-1} + 2 \approx 3.84$ ← abs max
3	$f(3) = 15e^{-3} + 2 \approx 2.75$

(2) $y = \dfrac{7x^2}{x+1}$ find abs min on $(-1, 5]$

$-1 < x \leq 5$

$$y' = \frac{14x(x+1) - 7x^2(1)}{(x+1)^2}$$

$$= \frac{14x^2 + 14x - 7x^2}{(x+1)^2}$$

$$= \frac{7x(x+2)}{(x+1)^2}$$

$y' = 0$ or y' DNE

$0 = 7x(x+2)$

$x = 0 \quad x = -2$

$0 = (x+1)^2$

$x = -1$

y is not defined a $x = -1$ so through it out.

Critical numbers : $0, -2$.

Since -2 is not in $(-1, 5]$ we can ignore it

x	$f(x)$
0	$f(0) = 0$ ← abs. min.
5	$f(5) = \dfrac{7 \cdot 5^2}{5+1} = 29.2$

③ $y = 1 - x^2 - 2x$ find abs max on $(-2, 0)$

1) find critical pts.

2) use 2nd deriv. test to find rel. max

3) choose largest rel. max.

$y' = 0 - 2x - 2$

$y' = 0$ or y' DNE never happens.

$-2x - 2 = 0$

$x = -1$

2nd der. test

$y'' = -2$, $y''(-1) = -2 \Rightarrow$ $x = -1$ is a rel. max

We choose rel. max w/ largest value, $x = -1$ ✓

abs. max.

HW 18 #7 | $f(x) = 8\ln(x^2+1)$ find infl. pts / concavity.

$$f'(x) = \frac{8}{(x^2+1)} \cdot (2x) = \frac{16x}{x^2+1}$$

$$f''(x) = \frac{16(x^2+1) - 16x(2x)}{(x^2+1)^2}$$

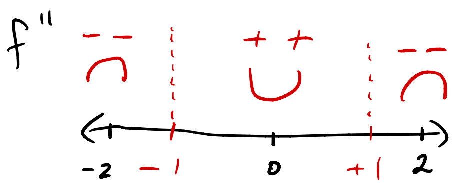
$$= \frac{16x^2+16-32x^2}{(x^2+1)^2}$$

$$= \frac{16(1-x^2)}{(x^2+1)^2}$$

$f'' = 0$ or f'' DNE

$0 = 16(1-x^2)$ $\qquad$ $0 = (x^2+1)^2$

$x = \pm 1$ $\qquad$ no solutions



$$f''(-2) = \frac{16(1-(-2)^2)}{((-2)^2+1)^2} < 0 \qquad f''(2) = \frac{16(1-(2)^2)}{(2^2+1)^2} < 0$$

$$f''(0) = \frac{16(1-0)}{(0^2+1)^2} > 0$$

So ± 1 are inflection pts.

Concave down $(-\infty, -1)$ and $(1, \infty)$

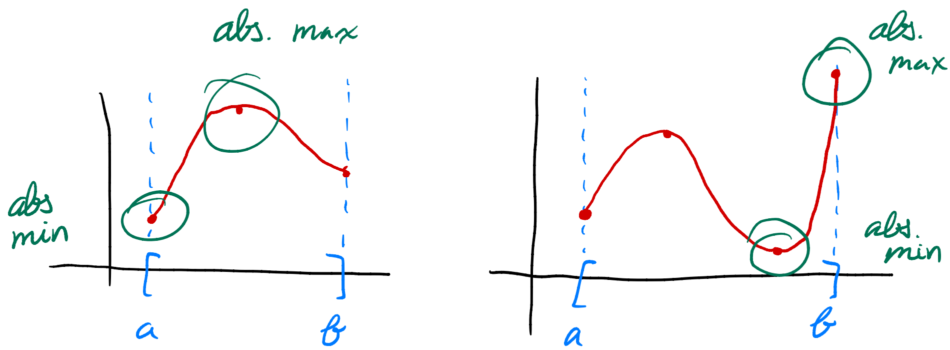
up $(-1, 1)$

Lesson 19: Absolute extrema on an interval

Let f be a function on an interval I

1) $x=c$ is an absolute max if $f(c)$ is the largest value of f on I.

2) $x=c$ is an absolute min if $f(c)$ is the smallest value of f on I.



How to find abs. extrema on $[a, b]$

1) find all critical numbers

2) Evaluate f at the critical numbers and a, b

3) Determine largest / smallest value.

Examples

① $y = \underbrace{5x}\underbrace{e^{-x}} + 2$ find abs. ext. on $[0, 3]$.

$y' = 5e^{-x} + 5x(-e^{-x}) + 0$

$= 5e^{-x}(1-x)$

$y' = 0$ or y' DNE never happens.

$0 = 5e^{-x}(1-x)$

e^{-x} is always positive (in particular never equal to 0)

$0 = 1 - x$

$x = 1 \leftarrow$ only critical.

x	$f(x) = 5xe^{-x} + 2$
0	$f(0) = 2 \leftarrow$ abs. min
1	$f(1) = 5e^{-1} + 2 \approx 3.84 \leftarrow$ abs. max
3	$f(3) = 15e^{-3} + 2 \approx 2.75$

② $y = \dfrac{7x^2}{x+1}$ find abs. min on $(-1, 5]$

$-1 < x \leq 5$

$$y' = \frac{14x(x+1) - 7x^2(1)}{(x+1)^2}$$

$$= \frac{14x^2 + 14x - 7x^2}{(x+1)^2}$$

$$= \frac{7x(x+2)}{(x+1)^2}$$

$y' = 0$ or y' DNE

$0 = 7x(x+2)$

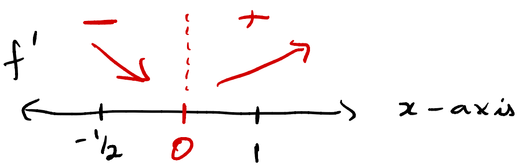
$x = 0 \quad x = -2$

-2 is not in $(-1, 5]$
so throw out -2.

$0 = (x+1)^2$

$x = -1$

Since y is not defined at $x = -1$ throw this pt out.



$$f'(-1/2) = \frac{7(-1/2)(-1/2+2)}{(-\frac{1}{2}+1)^2} < 0$$

$$f'(1) = \frac{7(1)(1+2)}{(1+1)^2} > 0$$

$x = 0$ is a relative min.

x	$f(x) = \dfrac{7x^2}{x+1}$
0	$f(0) = 0$ $\leftarrow$ abs min.
5	$f(5) = 29.2$