HL

$$
\begin{aligned}
\ln \left(\frac{e^{4}}{5 \sqrt{y^{3}}}\right) & =\ln \left(e^{4}\right)-\ln \left(5 \sqrt{y^{3}}\right) \\
& =\ln \left(e^{4}\right)-\ln \left(y^{3 / 5}\right) \\
& =4 \ln e-3 / 5 \ln y \\
& =4-3 / 5 \ln y
\end{aligned}
$$

Lesson 2: finding limits numerically / graphically What is a limit?
$f(x) \lim _{x \rightarrow c} f(x)$ is the limit of $f$ as $x$ approaches $c$
Numerically
(1) $f(x)=4 x+1, \lim _{x \rightarrow 2} f(x)=9$

| $x$ | 1.9 | 1.99 | 1.999 | 2 | 2.001 | 2.01 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 8.6 | 8.96 | 8.996 | 9 | 9.004 | 9.04 |

(2) $f(x)=\frac{-5}{x-1}, \lim _{x \rightarrow 1} f(x)=$ DNE

| $x$ | 9 | 99 | 999 | 1 | 1.001 | 1.01 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 50.1 | 500 | 5000 |  | -5000 | -500 |

(3) $f(x)=\left\{\begin{array}{ll}x^{2} & \text { if } x \neq 0 \\ 1 & \text { if } x=0\end{array}, \lim _{x \rightarrow 0} f(x)\right.$

| $x$ | -.1 | -.01 | 0 | .01 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| .01 | .0001 |  | .0001 | .01 |  |
|  | 000 | 0 |  |  |  |

then $\lim _{x \rightarrow 0} f(x)=0$, but $f(0)=1$
One-sided limits
"from the left" $=\lim _{x \rightarrow c^{-}} f(x)$
"from the right" $=\lim _{x \rightarrow c^{+}} f(x)$
$\lim _{x \rightarrow c} f(x)$ exists, when $\lim _{x \rightarrow c^{-}} f(x)=\lim _{x \rightarrow c^{+}} f(x)$
we let $\lim _{x \rightarrow c} f(x):=\lim _{x \rightarrow c^{-}} f(x)$
(4) $f(x)=\frac{\sin x}{x}, \lim _{x \rightarrow 0} f(x)$

| $x$ | -.1 | -.01 | 0 | .01 | .1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | .998 | .999 |  | .999 | .998 |

$$
\lim _{x \rightarrow 0^{-}} f(x)=1, \quad \lim _{x \rightarrow 0^{+}} f(x)=1
$$

$\Longrightarrow \lim _{x \rightarrow 0} f(x)=1$, but $f(0)=$ DNE

Evaluating limits Gnaphically

$$
\begin{aligned}
& \lim _{x \rightarrow c} f(x)=L^{x} \\
& \lim _{x \rightarrow c^{-}} f(x)=L_{-} \\
& \lim _{x \rightarrow c^{+}} f(x)=L_{+} \\
& c=-5^{-}: L=-2
\end{aligned}
$$

$$
{ }_{5}^{5} \left\lvert\, \begin{array}{lll} 
& \\
\lim _{x \rightarrow c^{-}} f(x) & f(x) & L_{-}
\end{array}\right.
$$

$$
\begin{array}{ll}
C=-3, & L_{-}=0, L_{+}=1.5, L=\text { DNE } \\
C=1, L_{-}=-\infty, L_{+}=+\infty, L=D N E
\end{array}
$$



$$
\begin{aligned}
& c=2 \\
& \lim _{x \rightarrow 2^{-}} f(x)=-\infty \\
& \lim _{x \rightarrow 2^{+}} f(x)=-\infty \\
& \lim _{x \rightarrow 2} f(x)=-\infty \\
& f(2)=\text { DNE }
\end{aligned}
$$

Office Hours. WTHR 313 (This is chemistry building) 9 will be there Mw: 3:30-4:30 pm
$H \omega 1 \# 6 / \csc \theta=5, \quad \frac{\pi}{2}<\theta<\pi$, find $\cos \theta$

$$
\begin{aligned}
\csc \theta=\frac{1}{\sin \theta} & =5 \quad, \quad \text { then } \sin \theta=1 / 5 \\
\sin ^{2} \theta+\cos ^{2} \theta=1 & \\
(1 / 5)^{2}+\cos ^{2} \theta & =1 \\
\cos ^{2} \theta & =1-1 / 25 \\
& =24 / 25 \\
\cos \theta & = \pm\left.\frac{\left(M M_{S}^{\pi / 2}\right.}{T}\right|_{C} ^{\pi} \\
& +\sqrt{24} / 5 \quad \cos \theta=-\sqrt{24} / 5
\end{aligned}
$$

Lesson 2: find limits numerically/graphricelly and One-sided limits.
What is a limit?
Consider as function $f(x)$
$\lim _{x \rightarrow c} f(x)=$ the limit of $f(x)$ as $x$ approaches $c$

Numerically
(1) $f(x)=4 x+1, \quad \lim _{x \rightarrow 2} f(x)$


$$
\lim _{x \rightarrow 2} f(x)=9
$$

(2) $f(x)=\frac{-5}{x-1}, \lim _{x \rightarrow 1} f(x)$

| $x$ | .9 | .99 | .999 | 1 | 1.001 | 1.01 | 1.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 50 | 500 | 5000 |  | -5000 | -500 | -50 |

$$
\begin{aligned}
& > \\
& D E
\end{aligned}
$$

(3) $f(x)= \begin{cases}x^{2} & \text { if } x \neq 0 \\ 1 & \text { if } x=0, \lim _{x \rightarrow 0} f(x)\end{cases}$

$$
f(0)=1 \quad \begin{array}{l|l|l|l|l|l}
x & -.1 & -.01 & 0 & .01 & 1 \\
\hline .01 & .0001 & & \underbrace{<0001}_{0} & .01
\end{array}
$$

$$
\lim _{x \rightarrow 0} f(x)=0, \text { but } f(0)=1
$$


$O_{n c}$ - sided limits
"from the left" $=\lim _{x \rightarrow 0^{-}} f(x)$

$$
\text { "from the rught" }=\lim _{x \rightarrow c^{+}} f(x)
$$

We say if $\lim _{x \rightarrow c^{-}} f(x)=\lim _{x \rightarrow c^{+}} f(x)$,
then $\lim _{x \rightarrow c} f(x)$ exists and is equal to the onc-sidad limits

$$
\begin{aligned}
& \text { (4) } f(x)=\frac{\sin x}{x}, \lim _{x \rightarrow 0} f(x) \\
& f(0)=\text { DUE, } \\
& \begin{array}{l|l|l|l|l|l}
x & -.1 & -.01 & 0 & .01 & .1 \\
\hline f(x) & .998 & .999 & & .999 & .998 \\
\end{array}
\end{aligned}
$$

$$
\lim _{x \rightarrow 0^{-}} f(x)=1, \lim _{x \rightarrow 0^{+}} f(x)=1
$$

So $\quad \lim _{x \rightarrow 0} f(x)=1$
Very important limit
find limits Graphically


$$
\begin{aligned}
& \lim _{x \rightarrow 1^{-}} f(x)=-\infty, \lim _{x \rightarrow 1^{+}} f(x)=+\infty \\
& \lim _{x \rightarrow 1^{-}} f(x)=\operatorname{DNE}
\end{aligned}
$$



$$
\begin{aligned}
& \lim _{x \rightarrow 2^{-}} f(x)=-\infty \\
& \lim _{x \rightarrow 2^{+}} f(x)=-\infty \\
& \lim _{x \rightarrow 2^{-}} f(x)=-\infty
\end{aligned}
$$

$$
\begin{aligned}
& f(1)=5, \operatorname{bot} \\
& \lim _{x \rightarrow 1^{-}} f(x)=0, \quad \lim _{x \rightarrow 1^{+}} f(x)=0, \lim _{x \rightarrow 1} f(x)=0
\end{aligned}
$$

