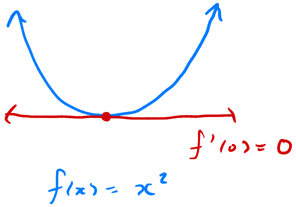
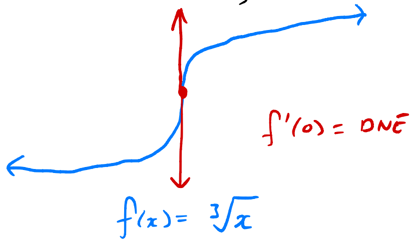
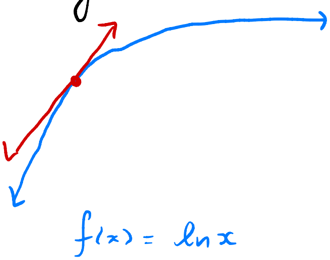
Lesson 20: Graphical interpretation of derivatives

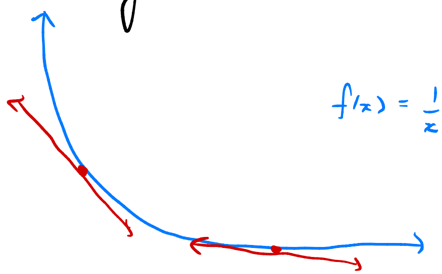
Critical numbers: $f'(x) = 0$ or $f'(x)$ DNE





Increasing $f'(x) > 0$ ↗ **Decreasing** $f'(x) < 0$ ↘

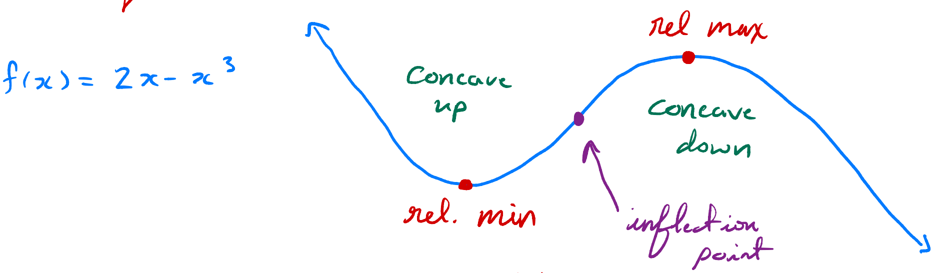




Relative max $x = c$ is critical number and $f''(c) < 0$.

Relative min $x = c$ is critical number and $f''(c) > 0$.

(first derivative switches signs at $x = c$)



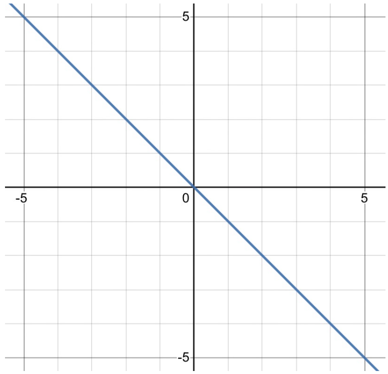
Concave up: $f''(x) > 0$ $\overset{++}{\cup}$

Concave down: $f''(x) < 0$ $\overset{--}{\cap}$

Inflection points $x = c$ such that $f''(c) = 0$ or DNE and f'' to switch signs at $x = c$

Examples

(1)



Critical numbers: none

Increasing: none

Decreasing: $(-\infty, \infty)$

Relative max: none

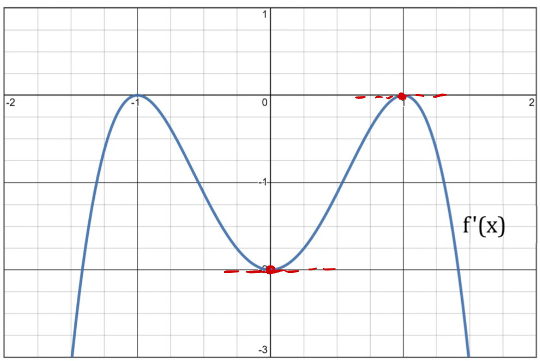
Relative min: none

Concave up: none

Concave down: none.

Inflection pts: none

(2)



Critical numbers

$x = -1$, $x = 1$

Increasing: none

Decreasing: $(-\infty, \infty)$

Relative extrema

none

Concave up. $f''(x) > 0$ we need $f'(x)$ to be inc.

$(-\infty, -1)$, $(0, 1)$

Concave down: $f''(x) < 0$ we need $f'(x)$ to be dec.

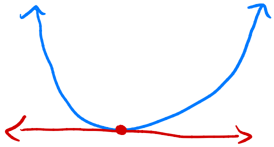
$(-1, 0)$, $(1, \infty)$

Inflection points: $f''(c) = 0$ or DNE f'' to switch signs

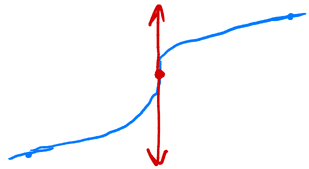
$x = -1, 0, 1$.

Lesson 20: Graphical interpretation of derivatives

Critical numbers: $x = c$ $\quad f'(c) = 0$ or DNE

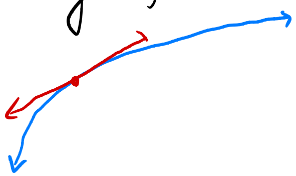


$f(x) = x^2$



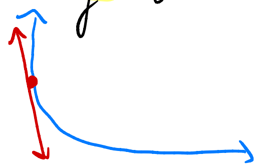
$f(x) = \sqrt[3]{x}$

Increasing: $f'(x) > 0$



$f(x) = \ln(x)$

Decreasing: $f'(x) < 0$

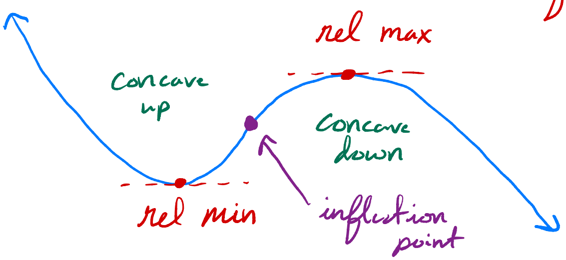


$f(x) = \frac{1}{x}$

Relative max: $x = c$ critical number and $f''(c) < 0$.

Relative min: $x = c$ critical number and $f''(c) > 0$.

(also could determine if f' switches signs near c)



$f(x) = 2x - x^3$

Concave up: $f''(x) > 0$ $\quad + +$

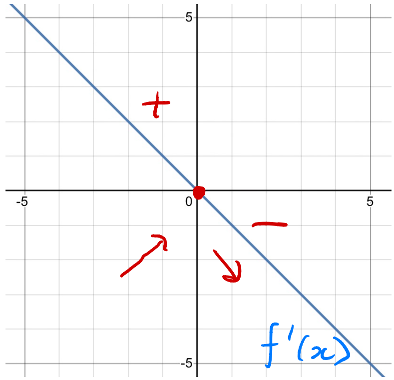
Concave down: $f''(x) < 0$ $\quad - -$

Inflection points: $x = c$ if $f''(c) = 0$ or DNE

and f'' to switch signs at $x = c$.

Examples

①



Points of f.

Critical numbers: $f'(x)=0$

$x=0$

Increasing: $f'(x)>0$

$(-\infty, 0)$

Decreasing: $f'(x)<0$

$(0, \infty)$

Relative max: $x=0$

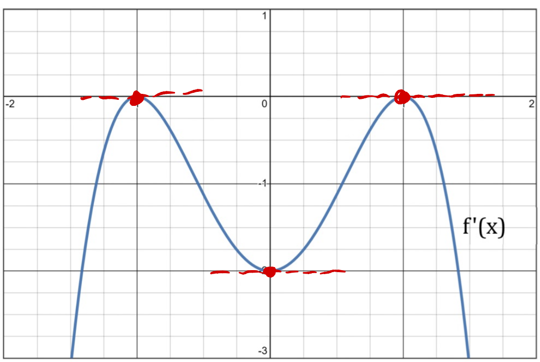
Relative min: none

Concave up: $f''(x)>0$, $f'(x)$ is increasing, none

Concave down: $f''(x)<0$, $f'(x)$ is decreasing, $(-\infty,\infty)$

Inflections: none.

②



Points for f

Critical numbers:

$x=-1, +1$

Increasing: $f'(x)>0$

none

Decreasing: $f'(x)<0$

$(-\infty, \infty)$

Relative extrema: none

Concave up: $f''(x)>0$, $f'(x)$ is inc. $(-\infty,-1)$, $(0,1)$

Concave down: $f''(x)<0$, $f'(x)$ is dec. $(-1,0)$, $(1,\infty)$

Inflection points: $f''(c)=0$ or DNE, f'' switch signs all $x = 1, 0, -1$ are enflection pts.